Published by Institute of Physics Publishing for SISSA

Received: *April 3, 2006*
Accepted: *April 20, 2006*
Published: *May 9, 2006*

D-branes and closed string field theory

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Abstract: We construct BRST invariant solitonic states in the OSp invariant string field theory for closed bosonic strings. Our construction is a generalization of the one given in the noncritical case. These states are made by using the boundary states for D-branes, and can be regarded as states in which D-branes or ghost D-branes are excited. We calculate the vacuum amplitude in the presence of solitons perturbatively and show that the cylinder amplitude for the D-brane is reproduced. The results imply that these are states with even number of D-branes or ghost D-branes.

Keywords: D-branes, Bosonic Strings, String Field Theory.

Contents

1. Introduction

D-branes have been studied for many years and used to reveal nonperturbative aspects of string theory. They are considered to be solitons in string theory. From the viewpoint of open string theory, (for example, in the vacuum string field theory [1]), D-branes emerge as soliton-like solutions of the equation of motion.

The question we would like to address in this paper is "what are D-branes in closed string field theory?". Although several attempts have been made [2, 3], D-branes have not been studied so much in the context of closed string field theories.

Actually, a fairly clear answer to the above question is given for noncritical strings. D-branes in noncritical string theories can be defined as in the critical ones [4]. In ref. [5], Fukuma and Yahikozawa showed that the D-branes can be realized as solitonic operators which commute with the Virasoro and W constraints [6] for the noncritical string theories. In ref. [7], it was shown that how such solitonic operators are realized in the string field theory of noncritical strings presented in ref. [8]. States in which D-branes are excited can be made by acting these solitonic operators on the vacuum.

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What we would like to do in this paper is to construct such states in a critical bosonic string field theory. Since the string field Hamiltonian given in ref. [8] consists of the joining-splitting interactions, it seems possible to generalize the construction of the operators given in ref. [7] to the string field theories with the light-cone gauge type interactions. In this paper, we take the OSp invariant string field theory [9] as such a theory. The OSp invariant string field theory is a covariantized version of the light-cone gauge string field theory [10] and it was proved that the S-matrix elements coincide with those of the light-cone gauge one [11].

What we will do is to construct solitonic operators made from the creation and annihilation operators of second-quantized strings. In order to deal with D-branes, we consider the closed strings whose wave functions are proportional to the boundary states. Such states were shown [12] to satisfy the idempotency equations. Because of these relations, we expect that three string vertices for such strings look quite like the ones which appear in the noncritical string field theory in ref. [8]. We will show that we can construct solitonic operators using such states. These operators can be considered as creation operators of D-branes [1] (or ghost D-branes recently proposed in ref. [14]). Acting them on the vacuum, we obtain BRST invariant states, which can be regarded as states in which D-branes are excited. We calculate the vacuum amplitude and show that these operators create two D-branes.

The organization of this paper is as follows. In section 2, we will briefly explain the construction of the solitonic operators [7] in the string field theory of noncritical strings [8]. In section 3, we will review the OSp invariant string field theory [9, 11]. In section 4, we will define the boundary states and the creation and annihilation operators for the strings whose wave functions are proportional to such states, in the OSp invariant string field theory. In section 5, we will construct solitonic operators using the operators defined in section 4, following the construction in the noncritical case. Regarding them as the creation operators for D-branes, we can get BRST invariant states in which D-branes are excited. We compute the vacuum amplitude and find that the operators we construct create two D-branes or ghost D-branes. Section 6 will be devoted to discussions. In the appendices, we present the details of the calculations to derive the BRST transformations for component fields of the string field which are proportional to the boundary states.

2. D-branes in noncritical string field theory

Noncritical strings can be described by the string field theory constructed in [8] and its generalizations. For simplicity, let us consider the $c = 0$ case.[2] In this case, the only reparametrization invariant quantity which can specify the state of a closed string is its length l. Therefore we define the creation and annihilation operators $\psi^\dagger(l), \psi(l)$ of the string with length l which satisfy

$$[\psi(l), \psi^\dagger(l')] = \delta(l - l') \ . \tag{2.1}$$

[1] In a recent paper[13], the author speculated about such operators from a quite different point of view.

[2] In this paper, we will follow the conventions of [7] which are different from those of [8].

The correlation functions can be calculated using the stochastic Hamiltonian

$$\begin{aligned} H &= \int_0^\infty dl_1 \int_0^\infty dl_2 (l_1+l_2)\psi^\dagger(l_1+l_2)\psi(l_1)\psi(l_2) \\ &\quad + g_s^2 \int_0^\infty dl_1 \int_0^\infty dl_2 l_1 l_2 \psi^\dagger(l_1)\psi^\dagger(l_2)\psi(l_1+l_2) \\ &\quad + \int_0^\infty dl \rho(l)\psi^\dagger(l) \\ &= \int_0^\infty dl \psi^\dagger(l)(lT(l)+\rho(l)) \ , \end{aligned} \tag{2.2}$$

where

$$\begin{aligned} T(l) &= \int_0^l dl' \psi(l')\psi(l-l') + g_s^2 \int_0^\infty dl' l' \psi^\dagger(l')\psi(l+l') \ , \\ \rho(l) &= 3\delta''(l) - \frac{3}{4}\mu\delta(l) \ . \end{aligned} \tag{2.3}$$

The processes which the first two terms in the Hamiltonian represent are exactly the joining-splitting interactions. The third term corresponds to a tadpole term in which only strings with vanishing length are involved. Here g_s denotes the string coupling constant and μ denotes the cosmological constant on the worldsheet.

In this formulation, the Virasoro constraint for $c = 0$ string theory can be written as

$$T(l)|\Psi\rangle = 0 \ , \tag{2.4}$$

where $|\Psi\rangle$ is a state which can be expressed by using the correlation functions. Solitonic operators corresponding to D-branes can be constructed as operators which commute with $T(l)$ [5]. If such operators exist, by acting them on $|\Psi\rangle$ which is a solution of eq. (2.4), one can generate other solutions.

From the commutation relations

$$\begin{aligned} \left[\frac{1}{g_s^2}\int_0^\infty dl' l' \epsilon(l') T(l') \, , \, \frac{1}{g_s l}\psi(l)\right] &= -\frac{1}{g_s}\int_0^\infty dl' l' \epsilon(l')\psi(l+l') \ , \\ \left[\frac{1}{g_s^2}\int_0^\infty dl' l' \epsilon(l') T(l') \, , \, g_s\psi^\dagger(l)\right] &= g_s \int_0^l dl' l' (l-l')\epsilon(l')\psi^\dagger(l-l') \\ &\quad + \frac{2}{g_s}\int_0^\infty dl' (l+l')\epsilon(l+l')\psi(l') \ , \end{aligned} \tag{2.5}$$

it is straightforward to show that

$$\mathcal{V}(\zeta) \equiv \exp\left(g_s \int_0^\infty dl e^{-\zeta l}\psi^\dagger(l)\right) \exp\left(-\frac{2}{g_s}\int_0^\infty \frac{dl}{l} e^{\zeta l}\psi(l)\right) \tag{2.6}$$

satisfies

$$\left[\frac{1}{g_s^2}\int_0^\infty dl' l' \epsilon(l') T(l') \, , \, \mathcal{V}(\zeta)\right] = \partial_\zeta \left(\partial_\zeta \tilde{\epsilon}(\zeta)\mathcal{V}(\zeta)\right) \ , \tag{2.7}$$

where

$$\tilde{\epsilon}(\zeta) = \int_0^\infty dl e^{-\zeta l}\epsilon(l) \ . \tag{2.8}$$

Therefore $\int d\zeta \mathcal{V}(\zeta)$ commutes with $T(l)$ if the limits of the integral are chosen appropriately. In perturbative calculations, the integration over ζ is done by the saddle point method and we do not have to specify these limits. This operator can be identified with the creation operator of the ZZ-brane. $\psi(l)$ in the exponent in $\mathcal{V}(\zeta)$ has the effect of generating boundaries on the worldsheet with exactly the same weight as that for the boundary state of the ZZ-brane. Moreover one can see that the solitonic operator increases the number of the eigenvalues of the matrix for the matrix model, by one.

The calculations above are quite analogous to the ones in 2D free boson theory. ψ, $\psi^\dagger$ and $T(l)$ can be compared to the oscillator modes of the boson and its energy-momentum tensor respectively. $\mathcal{V}(\zeta)$ should correspond to the vertex operator with conformal weight 1. The condition that the right hand side of eq. (2.7) be a total derivative fixes the overall factor in the exponent of $\mathcal{V}(\zeta)$. Actually, from this condition alone, there exists another choice for $\mathcal{V}(\zeta)$ which is

$$\exp\left(-g_s \int_0^\infty dl e^{-\zeta l}\psi^\dagger(l)\right)\exp\left(\frac{2}{g_s}\int_0^\infty \frac{dl}{l}e^{\zeta l}\psi(l)\right) . \tag{2.9}$$

This operator should correspond to the ghost D-brane.

What we would like to do in this paper is to generalize the above construction to critical strings. Since the solitonic operator $\mathcal{V}(\zeta)$ generates boundaries on the worldsheet, we should use the creation and annihilation operators of the critical strings whose wave functions are proportional to the boundary states, in place of $\psi^\dagger$ and ψ in the above construction. In ref. [12], it was shown that the boundary states $|B\rangle$ satisfy the idempotency equation

$$|B\rangle * |B\rangle \propto |B\rangle, \tag{2.10}$$

where $*$ denotes the product corresponding to a light-cone gauge type three string vertex. This equation implies that in the joining-splitting interaction for the strings whose wave function is proportional to the boundary states, what matter are only their lengths. Therefore the three string vertex for such states is essentially the same as the one in the Hamiltonian (2.2) for noncritical strings. Hence it seems likely that the above construction works also for some string field theory of critical strings.

3. OSp invariant string field theory

The string field theory we consider in this paper is the OSp invariant string field theory [9, 11]. In order to fix the notations used in this paper, let us recapitulate the formulation of this theory.

OSp extension. Siegel's procedure [9] for covariantizing the light-cone gauge string field theory [10, 15][16] is to replace the $O(24)$ transverse vector X^i by the $OSp(25,1|2)$ vector $X^M = \left(\sqrt{\frac{2}{\alpha'}}X^\mu, C, \bar{C}\right)$, where $X^\mu = (X^i, X^+, X^-)$ are Grassmann even and the ghost

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fields C and $\bar{C}$ are Grassmann odd. The metric of the $OSp(25,1|2)$ vector space is

$$\eta_{MN} = \begin{array}{c} \\ \\ C \\ \bar{C} \end{array} \!\!\begin{array}{c} \begin{array}{cc} \quad\quad\quad & C \;\; \bar{C} \end{array} \\ \left(\begin{array}{c|cc} \eta_{\mu\nu} & & \\ \hline & 0 & -i \\ & i & 0 \end{array} \right) \end{array} = \eta^{MN} \ . \tag{3.1}$$

The Euclidean action is

$$S = \frac{1}{8\pi} \int d\tau d\sigma \partial_a X^M \partial^a X^N \eta_{MN} \ , \tag{3.2}$$

where (τ, σ) denote the coordinates on the cylinder worldsheet. One has the mode expansion

$$\begin{aligned} X^\mu(\tau,\sigma) &= x^\mu - \alpha' i p^\mu \tau + i\sqrt{\frac{\alpha'}{2}} \sum_{n\neq 0} \frac{1}{n} \left(\alpha_n^\mu e^{-n(\tau+i\sigma)} + \tilde{\alpha}_n^\mu e^{-n(\tau-i\sigma)} \right) \ , \\ C(\tau,\sigma) &= C_0 + 2i\pi_0\tau - i\sum_{n\neq 0} \frac{1}{n} \left(\gamma_n e^{-n(\tau+i\sigma)} + \tilde{\gamma}_n e^{-n(\tau-i\sigma)} \right) \ , \\ \bar{C}(\tau,\sigma) &= \bar{C}_0 - 2i\bar{\pi}_0\tau + i\sum_{n\neq 0} \frac{1}{n} \left(\bar{\gamma}_n e^{-n(\tau+i\sigma)} + \tilde{\bar{\gamma}}_n e^{-n(\tau-i\sigma)} \right) \ . \end{aligned} \tag{3.3}$$

The nonvanishing canonical commutation relations are

$$\begin{aligned} &[x^\mu, p^\nu] = i\eta^{\mu\nu} \ , \quad [\alpha_n^\mu, \alpha_m^\nu] = n\eta^{\mu\nu}\delta_{n+m,0} \ , \quad [\tilde{\alpha}_n^\mu, \tilde{\alpha}_m^\nu] = n\eta^{\mu\nu}\delta_{n+m,0} \ , \\ &\{C_0, \bar{\pi}_0\} = 1 \ , \quad \{\bar{C}_0, \pi_0\} = 1 \ , \quad \{\gamma_n, \bar{\gamma}_m\} = in\delta_{n+m,0} \ , \quad \{\tilde{\gamma}_n, \tilde{\bar{\gamma}}_m\} = in\delta_{n+m,0} \end{aligned} \tag{3.4}$$

for $n \neq 0$. We also use

$$\alpha_0^\mu = \tilde{\alpha}_0^\mu = \sqrt{\frac{\alpha'}{2}} p^\mu \ , \quad \gamma_0 = \tilde{\gamma}_0 \equiv \pi_0 \ , \quad \bar{\gamma}_0 = \tilde{\bar{\gamma}}_0 \equiv \bar{\pi}_0 \ . \tag{3.5}$$

The Hilbert space for the string consists of the Fock space of the oscillators and the wave function for the zero modes. We take the wave function to be a function of $p^\mu, \alpha, \pi_0, \bar{\pi}_0$, i.e. we take the momentum representation for the zero modes. Here α denotes the string length, which is a variable characteristic of the string field theories with the joining-splitting interactions. In the momentum representation, the vacuum state $|0\rangle$ in the first quantization is defined by

$$\begin{aligned} &\alpha_n^\mu|0\rangle = \tilde{\alpha}_n^\mu|0\rangle = 0 \ , \quad \gamma_n|0\rangle = \tilde{\gamma}_n|0\rangle = 0 \ , \quad \bar{\gamma}_n|0\rangle = \tilde{\bar{\gamma}}_n|0\rangle = 0 \qquad \text{for } n > 0 \ , \\ &x^\mu|0\rangle = i\frac{\partial}{\partial p_\mu}|0\rangle = 0 \ , \quad C_0|0\rangle = \frac{\partial}{\partial \bar{\pi}_0}|0\rangle = 0 \ , \quad \bar{C}_0|0\rangle = \frac{\partial}{\partial \pi_0}|0\rangle = 0 \ , \\ &\frac{\partial}{\partial \alpha}|0\rangle = 0 \ . \end{aligned} \tag{3.6}$$

The integration measure for the zero modes of the r-th string is written as

$$dr \equiv (2\pi)^{-27}\alpha_r d\alpha_r d^{26}p_r \, id\bar{\pi}_0^{(r)} d\pi_0^{(r)} \ . \tag{3.7}$$

The BRST charge is defined [17, 18] as

$$\begin{aligned} Q_{\mathrm{B}} &= \frac{C_0}{2\alpha}(L_0+\tilde{L}_0-2) - i\pi_0\frac{\partial}{\partial\alpha} \\ &\quad + \frac{i}{\alpha}\sum_{n=1}^{\infty}\left(\frac{\gamma_{-n}L_n - L_{-n}\gamma_n}{n} + \frac{\tilde{\gamma}_{-n}\tilde{L}_n - \tilde{L}_{-n}\tilde{\gamma}_n}{n}\right) , \end{aligned} \tag{3.8}$$

where L_n and $\tilde{L}_n$ ($n \in \mathbb{Z}$) are the Virasoro generators defined as

$$\begin{aligned} L_n &\equiv \sum_m \circ\!\!\circ \left(\frac{1}{2}\alpha^{\mu}_{n+m}\alpha_{-m,\mu} + i\gamma_{n+m}\bar{\gamma}_{-m}\right)\circ\!\!\circ \ , \\ \tilde{L}_n &\equiv \sum_m \circ\!\!\circ \left(\frac{1}{2}\tilde{\alpha}^{\mu}_{n+m}\tilde{\alpha}_{-m,\mu} + i\tilde{\gamma}_{n+m}\tilde{\bar{\gamma}}_{-m}\right)\circ\!\!\circ \ . \end{aligned} \tag{3.9}$$

Here $\circ\!\!\circ \ \circ\!\!\circ$ means the normal ordering of the oscillators in which the non-negative modes should be moved to the right of the negative modes. The BRST charge (3.8) is nilpotent: $(Q_{\mathrm{B}})^2 = 0$.

The reflector The reflector is defined as

$$\langle R(1,2)| = \delta_{\mathrm{LC}}(1,2)\ {}_{12}\langle 0|\, e^{E(1,2)}\,\frac{1}{\alpha_1} \ , \tag{3.10}$$

where

$$\begin{aligned} {}_{12}\langle 0| &= {}_1\langle 0|{}_2\langle 0| \ , \\ E(1,2) &= -\sum_{n=1}^{\infty}\frac{1}{n}\left(\alpha_n^{M(1)}\alpha_n^{N(2)} + \tilde{\alpha}_n^{M(1)}\tilde{\alpha}_n^{N(2)}\right)\eta_{MN} \ , \\ \delta_{\mathrm{LC}}(1,2) &= i(2\pi)^{27}\delta(\alpha_1+\alpha_2)\delta^{26}(p_1+p_2)(\bar{\pi}_0^{(1)}+\bar{\pi}_0^{(2)})(\pi_0^{(1)}+\pi_0^{(2)}) \ , \end{aligned} \tag{3.11}$$

with

$$\alpha_n^M = (\alpha_n^{\mu}, -\gamma_n, \bar{\gamma}_n) \ , \quad \tilde{\alpha}_n^M = (\tilde{\alpha}_n^{\mu}, -\tilde{\gamma}_n, \tilde{\bar{\gamma}}_n) \ . \tag{3.12}$$

We also introduce

$$|R(1,2)\rangle = \delta_{\mathrm{LC}}(1,2)\frac{1}{\alpha_1}e^{E^{\dagger}(1,2)}|0\rangle_{12} \ . \tag{3.13}$$

The reflector $\langle R(1,2)|$ satisfies

$$\begin{aligned} &\langle R(1,2)|\,(\alpha_1+\alpha_2) = 0 \ , \ \ \langle R(1,2)|\left(x^{\mu(1)} - x^{\mu(2)}\right) = 0 \ , \\ &\langle R(1,2)|\left(C_0^{(1)} - C_0^{(2)}\right) = 0 \ , \ \ \langle R(1,2)|\left(\bar{C}_0^{(1)} - \bar{C}_0^{(2)}\right) = 0 \ , \\ &\langle R(1,2)|\left(\alpha_n^{M(1)} + \alpha_{-n}^{M(2)}\right) = 0 \ , \ \ \langle R(1,2)|\left(\tilde{\alpha}_n^{M(1)} + \tilde{\alpha}_{-n}^{M(2)}\right) = 0 \quad \text{for } \forall n \in \mathbb{Z} \ . \end{aligned} \tag{3.14}$$

This yields

$$\langle R(1,2)| \left(L_n^{(1)} - L_{-n}^{(2)} \right) = 0 \ , \quad \langle R(1,2)| \left(\tilde{L}_n^{(1)} - \tilde{L}_{-n}^{(2)} \right) = 0 \quad \text{for } \forall n \in \mathbb{Z}$$
$$\langle R(1,2)| \left(Q_B^{(1)} + Q_B^{(2)} \right) = 0 \ . \tag{3.15}$$

$|R(1,2)\rangle$ satisfies similar identities.

The BPZ conjugate $\langle\Phi|$ of $|\Phi\rangle$ is defined as

$$_2\langle\Phi| = \int d1 \, \langle R(1,2)|\Phi\rangle_1 \ . \tag{3.16}$$

From the definitions, we have

$$\int d1d2 \, \langle R(1,2)|\Phi\rangle_1 |\Psi\rangle_2 = -(-1)^{|\Phi||\Psi|} \int d1d2 \, \langle R(1,2)|\Psi\rangle_1 |\Phi\rangle_2 \ , \tag{3.17}$$

and

$$\int d1 \, {}_1\langle\Phi|R(1,2)\rangle = |\Phi\rangle_2 \ , \tag{3.18}$$

where $(-1)^{|\Phi|}$ denotes the Grassmann parity of the string field Φ. Thus $\langle R(1,2)|$ can be considered as the symplectic form for the string fields and $|R(1,2)\rangle$ is its inverse.

The three string vertex. The three string vertex is given by

$$\begin{aligned}
\langle V_3(1,2,3)| &= \delta_{\mathrm{LC}}(1,2,3) \ {}_{123}\langle 0| e^{E(1,2,3)} C(\rho_I) \mathcal{P}_{123} \frac{|\mu(1,2,3)|^2}{\alpha_1\alpha_2\alpha_3} \\
&= i\delta(1,2,3) \ {}_{123}\langle 0| e^{E(1,2,3)} \left(\sum_{r=1}^{3} \bar{\pi}_0^{(r)} \right) \left(\sum_{s=1}^{3} \pi_0^{(s)} \right) C(\rho_I) \mathcal{P}_{123} \frac{|\mu(1,2,3)|^2}{\alpha_1\alpha_2\alpha_3} \ ,
\end{aligned} \tag{3.19}$$

where ρ_I denotes the interaction point and

$$\begin{aligned}
{}_{123}\langle 0| &= {}_1\langle 0| \, {}_2\langle 0| \, {}_3\langle 0| \ , \\
\mathcal{P}_{123} &= \mathcal{P}_1 \mathcal{P}_2 \mathcal{P}_3 \ , \quad \mathcal{P}_r = \int_0^{2\pi} \frac{d\theta}{2\pi} e^{i\theta \left(L_0^{(r)} - \tilde{L}_0^{(r)} \right)} \ , \\
\delta_{\mathrm{LC}}(1,2,3) &= i(2\pi)^{27} \, \delta^{26} \left(\sum_{r=1}^{3} p_r \right) \delta \left(\sum_{s=1}^{3} \alpha_s \right) \left(\sum_{r'=1}^{3} \bar{\pi}_0^{(r')} \right) \left(\sum_{s'=1}^{3} \pi_0^{(s')} \right) \ , \\
\delta(1,2,3) &= (2\pi)^{27} \, \delta^{26} \left(\sum_{r=1}^{3} p_r \right) \delta \left(\sum_{s=1}^{3} \alpha_s \right) \ , \\
E(1,2,3) &= \sum_{n,m\geq 0} \sum_{r,s} \bar{N}_{nm}^{rs} \left(\frac{1}{2} \alpha_n^{\mu(r)} \alpha_{m\mu}^{(s)} + i\gamma_n^{(r)} \bar{\gamma}_m^{(s)} + \frac{1}{2} \tilde{\alpha}_n^{\mu(r)} \tilde{\alpha}_{m\mu}^{(s)} + i\tilde{\gamma}_n^{(r)} \tilde{\bar{\gamma}}_m^{(s)} \right) \ . \\
\mu(1,2,3) &= \exp \left(-\hat{\tau}_0 \sum_{r=1}^{3} \frac{1}{\alpha_r} \right) \ , \quad \hat{\tau}_0 = \sum_{r=1}^{3} \alpha_r \ln |\alpha_r| \ .
\end{aligned} \tag{3.20}$$

Here $\bar{N}_{nm}^{rs}$ denote the Neumann coefficients associated with the joining-splitting type of three string interaction [10][15, 16].[3]

[3] Although the Neumann coefficients in the anti-holomorphic sector are complex conjugate to those in the holomorphic sector in general, one may choose the Neumann coefficients for the three string vertex to be real because of the $SL(2,\mathbb{C})$ invariance on the worldsheet.

By using the three string vertex (3.19), the $*$-product $\Phi * \Psi$ of two arbitrary closed string fields Φ and Ψ is defined by

$$|\Phi * \Psi\rangle_4 = \int d1d2d3 \; \langle V_3(1,2,3)\,|\Phi\rangle_1 \,|\Psi\rangle_2 \,|R(3,4)\rangle \; . \tag{3.21}$$

The $*$-product has following properties,

$$\begin{aligned} &Q_{\mathrm{B}}\,(\Phi * \Psi) = (Q_{\mathrm{B}}\Phi) * \Psi + (-1)^{|\Phi|}\Phi * (Q_{\mathrm{B}}\Psi) \; , \\ &(\Phi_1 * \Phi_2) * \Phi_3 + (-1)^{|\Phi_1|(|\Phi_2|+|\Phi_3|)}\,(\Phi_2 * \Phi_3) * \Phi_1 \\ &\qquad\qquad + (-1)^{|\Phi_3|(|\Phi_1|+|\Phi_2|)}\,(\Phi_3 * \Phi_1) * \Phi_2 = 0 \; . \end{aligned} \tag{3.22}$$

The first equation is equivalent to

$$\langle V_3(1,2,3)| \sum_{r=1}^{3} Q_{\mathrm{B}}^{(r)} = 0 \; , \tag{3.23}$$

and the second one is known as the Jacobi identity.

String field action The action of the OSp invariant string field theory is directly given by the OSp extension from that of the light-cone gauge string field theory. This takes the form

$$\begin{aligned} S = \int dt \Bigg[&\frac{1}{2}\int d1d2 \; \langle R(1,2)\,|\Phi\rangle_1 \left(i\frac{\partial}{\partial t} - \frac{L_0^{(2)} + \tilde{L}_0^{(2)} - 2}{\alpha_2} \right) |\Phi\rangle_2 \\ &+\frac{2g}{3}\int d1d2d3 \; \langle V_3(1,2,3)| \left(\sum_{r=1}^{3} \bar{\pi}_0^{(r)} \right) |\Phi\rangle_1 |\Phi\rangle_2 |\Phi\rangle_3 \Bigg] \; , \end{aligned} \tag{3.24}$$

where t denotes the proper time. The string field Φ is taken to be Grassmann even and subject to the level matching condition $\mathcal{P}\Phi = \Phi$. Note that in the interaction term the three string vertex $\langle V_3(1,2,3)|$ is multiplied by the factor $\sum_{r=1}^{3} \bar{\pi}_0^{(r)}$. This manipulation removes $C(\rho_I)$ from the vertex $\langle V_3(1,2,3)|$, i.e.

$$\left\langle V_3^0(1,2,3)\right| \equiv \langle V_3(1,2,3)| \left(\sum_{r=1}^{3} \bar{\pi}_0^{(r)} \right) = \delta_{\mathrm{LC}}(1,2,3) \;{}_{123}\langle 0| e^{E(1,2,3)} \mathcal{P}_{123} \frac{|\mu(1,2,3)|^2}{\alpha_1\alpha_2\alpha_3} \tag{3.25}$$

The action (3.24) is invariant under the BRST transformation

$$\delta_{\mathrm{B}}\Phi = Q_{\mathrm{B}}\Phi + g\Phi * \Phi \; , \tag{3.26}$$

where the $*$-product is defined in eq. (3.21). The nilpotency of the BRST transformation (3.26) is assured by the nilpotency of Q_{B} and eqs. (3.22). One can readily show that the action (3.24) is invariant under the BRST transformation (3.26) by using the nilpotency of the BRST transformation (3.26) and the fact that the action (3.24) can be expressed as

$$S = \int dt \left[\frac{1}{2}\int d1d2 \langle R(1,2)|\Phi\rangle_1 i\frac{\partial}{\partial t}|\Phi\rangle_2 + \delta_{\mathrm{B}}\left(\int d1d2 \langle R(1,2)|\Phi\rangle_1 \bar{\pi}_0^{(2)} |\Phi\rangle_2 \right) \right] \; . \tag{3.27}$$

In this string field theory, C and $\bar{C}$ play the role of the b, c ghost in the usual theory. Indeed with the following identifications

$$\begin{aligned}
&\gamma_n = in\alpha c_n \ , \quad \tilde{\gamma}_n = in\alpha\tilde{c}_n \ ; \quad \bar{\gamma}_n = \frac{1}{\alpha}b_n \ , \quad \tilde{\bar{\gamma}}_n = \frac{1}{\alpha}\tilde{b}_n \ , \\
&C_0 = 2\alpha c_0^+ \ , \quad \bar{\pi}_0 = \frac{1}{2\alpha}b_0^+ \ ,
\end{aligned} \tag{3.28}$$

with $n \neq 0$, Q_B becomes almost the same as the usual first-quantized BRST operator. Perturbative calculations can be done in a way similar to the one for the light-cone gauge string field theory. In the canonical quantization, we should think of the components of $|\Phi\rangle$ with positive α as annihilation operators and those with negative α as creation operators. The prescription for how to treat the physical on-shell states was given by ref. [11][4], and a proof was given to the fact that the S-matrix elements calculated using this theory coincide with those of the light-cone gauge string field theory.

Before concluding this section, one comment is in order. In refs.[19, 20][11, 21], gauge invariant actions were proposed and it was shown that the OSp invariant theory can be obtained from them after gauge fixing. Unfortunately the BRST transformations which originate from these covariantized light-cone string field theories coincide with eq. (3.26) only for on-shell states. In this paper, we should deal with the boundary states which are off-shell, and consider eq. (3.26) as the BRST transformation. The origin of this BRST transformation eq. (3.26) may be understood by considering this system in terms of the BFV formalism. In principle, looking at the BRST transformation itself, one can read off the constraints from which the BRST transformation is constructed.

4. Boundary state and string field

In order to construct solitonic operators in the way mentioned at the end of section 2, we should study the boundary states in the OSp invariant theory and identify the creation and annihilation operators corresponding to such states. A problem is that the boundary states are not normalizable. We will introduce a BRST invariant regularization and define normalizable states proportional to the boundary states.

In what follows, we consider the toroidally compactified space-time characterized by $X^\mu \simeq X^\mu + 2\pi R^\mu$ ($\mu = 0, 1, \ldots, 25$) to regularize the infrared divergence. In this situation, the zero modes of the matter sector are modified because of the momentum quantization and the windings. We briefly summarize the notations for the zero-mode part of the toroidally compactified matter sector.

The zero-mode part of $X^\mu(\tau, \sigma)$ takes the form

$$X^\mu(\tau,\sigma)\Big|_{\text{zero-mode}} = x_0^\mu + \alpha'\left(-ip^\mu\tau + q^\mu\sigma\right) = x_L^\mu + x_R^\mu - i\frac{\alpha'}{2}\left(p_L^\mu \ln w + p_R^\mu \ln \bar{w}\right) \ , \tag{4.1}$$

where w and $\bar{w}$ are complex coordinates on the cylinder worldsheet defined as $w = e^{\tau + i\sigma}$ and $\bar{w} = e^{\tau - i\sigma}$. The center-of-mass momentum p^μ is quantized and q^μ is related to the

[4] While the author of ref. [11] gives the prescription in the context of the gauge invariant covariantized light-cone string field theory, his prescription is applicable to the OSp invariant string field theory as well.

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winding number $\mathtt{w}^\mu$ as follows:

$$p^\mu = \frac{\mathtt{n}^\mu}{R^\mu} \ , \quad q^\mu = \frac{R^\mu \mathtt{w}^\mu}{\alpha'} \ , \qquad \mathtt{n}^\mu,\ \mathtt{w}^\mu \in \mathbb{Z}. \tag{4.2}$$

$x^\mu_{L,R}$ and $p^\mu_{L,R}$ are defined as

$$\begin{aligned} x_0^\mu = x_L^\mu + x_R^\mu \ , \ \ p_L^\mu &= p + q = \sqrt{\frac{2}{\alpha'}}\alpha_0^\mu = \left(\frac{\mathtt{n}^\mu}{R^\mu} + \frac{R^\mu \mathtt{w}^\mu}{\alpha'}\right) \ , \\ p_R^\mu &= p - q = \sqrt{\frac{2}{\alpha'}}\tilde{\alpha}_0^\mu = \left(\frac{\mathtt{n}^\mu}{R^\mu} - \frac{R^\mu \mathtt{w}^\mu}{\alpha'}\right) \ . \end{aligned} \tag{4.3}$$

They obey the canonical commutation relations: $\left[x_L^\mu\,,\,p_L^\nu\right] = \left[x_R^\mu\,,\,p_R^\nu\right] = i\eta^{\mu\nu}$, otherwise vanishing. Thus the zero-mode sector consists of two canonical pairs. For later use, we introduce a new variable $y_0^\mu \equiv x_L^\mu - x_R^\mu$ and we choose the basis of the zero-mode phase space to be $\{x_0^\mu, y_0^\mu; p^\mu, q^\mu\}$. They satisfy $[x_0^\mu\,,\,p^\nu] = [y_0^\mu\,,\,q^\nu] = i\eta^{\mu\nu}$. Because of the quantization (4.2) of q^μ, the range in which y_0^μ (conjugate to q^μ) varies is finite as well as that of x_0^μ:

$$0 \le x_0^\mu \le 2\pi R^\mu \ , \quad 0 \le y_0^\mu \le \frac{2\pi\alpha'}{R^\mu} \ . \tag{4.4}$$

Let $|x^\mu\rangle$ and $|y^\mu\rangle$ be the eigenstates of the operators x_0^μ and y_0^μ with eigenvalues x^μ and y^μ. Let $|\mathtt{n}^\mu\rangle$ and $|\mathtt{w}^\mu\rangle$ be the eigenstates of the operators p^μ and q^μ with eigenvalues $p^\mu = \frac{\mathtt{n}^\mu}{R^\mu}$ and $q^\mu = \frac{R^\mu \mathtt{w}^\mu}{\alpha'}$. We normalize these states as follows:

$$\begin{aligned} &\left\langle x'^\mu \,|x^\mu\right\rangle = \delta\left(x'^\mu - x^\mu\right) \ , \quad \left\langle y'^\mu \,|y^\mu\right\rangle = \delta\left(y'^\mu - y^\mu\right) \ , \\ &\left\langle \mathtt{n}'^\mu \,|\mathtt{n}^\mu\right\rangle = \delta_{\mathtt{n}'^\mu,\mathtt{n}^\mu} \ , \quad \left\langle \mathtt{w}'^\mu \,|\mathtt{w}^\mu\right\rangle = \delta_{\mathtt{w}'^\mu,\mathtt{w}^\mu} \ . \end{aligned} \tag{4.5}$$

It follows from eqs. (4.4) and (4.5) that

$$|x^\mu\rangle = \frac{1}{\sqrt{2\pi R^\mu}} \sum_{\mathtt{n}^\mu \in \mathbb{Z}} e^{-i\frac{\mathtt{n}^\mu}{R^\mu}}\,|\mathtt{n}^\mu\rangle \ , \quad |y^\mu\rangle = \sqrt{\frac{R^\mu}{2\pi\alpha'}} \sum_{\mathtt{w}^\mu \in \mathbb{Z}} e^{-i\frac{R^\mu \mathtt{w}^\mu}{\alpha'}y^\mu}\,|\mathtt{w}^\mu\rangle \ . \tag{4.6}$$

In accordance with the modification above, the momentum dependent parts of the integration measure for the zero modes, the reflector $\langle R(1,2)|$ and the three string vertex $\langle V_3(1,2,3)|$ are respectively replaced as follows:

$$\text{measure}: \ \int \frac{d^{26}p}{(2\pi)^{26}} \longrightarrow \prod_{\mu=0}^{25}\left(\sum_{\mathtt{n}^\mu\in\mathbb{Z}}\sum_{\mathtt{w}^\mu\in\mathbb{Z}}\right) \ , \tag{4.7}$$

$$\langle R(1,2)|: \ (2\pi)^{26}\delta^{26}(p_1+p_2) \longrightarrow \prod_{\mu=0}^{25}\left(\delta_{\mathtt{n}_1^\mu+\mathtt{n}_2^\mu,0}\,\delta_{\mathtt{w}_1^\mu+\mathtt{w}_2^\mu,0}\right) \ ,$$

$$\langle V_3(1,2,3)|: \ (2\pi)^{26}\delta^{26}\left(p_1+p_2+p_3\right) \longrightarrow \prod_{\mu=0}^{25}\left(\delta_{\mathtt{n}_1^\mu+\mathtt{n}_2^\mu+\mathtt{n}_3^\mu,0}\,\delta_{\mathtt{w}_1^\mu+\mathtt{w}_2^\mu+\mathtt{w}_3^\mu,0}\right) e^{-i\pi(\mathtt{n}_3\cdot\mathtt{w}_2-\mathtt{n}_1\cdot\mathtt{w}_1)} \ .$$

In the last equation, the cocycle factor $e^{-i\pi(\mathtt{n}_3\cdot\mathtt{w}_2-\mathtt{n}_1\cdot\mathtt{w}_1)}$ is necessary for the Jacobi identity to be satisfied [22, 23]. (See also the second paper in ref. [24].)

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4.1 Boundary state

We consider the situation in which the Dp-branes extend in the x^μ directions with $\mu = 0, 1, \ldots, p$. We refer to these directions as the Neumann directions and denote them by x^μ ($\mu \in \mathrm{N}$). We refer to the directions transverse to the D-branes as the Dirichlet directions and denote them by x^i ($i \in \mathrm{D}$). In the first quantized approach to the closed string, the D-brane is described by the boundary state.

The boundary state in the matter sector $\left|B^X\right\rangle$ is expressed as the direct product of those for the Neumann and the Dirichlet sectors:

$$\begin{aligned} \left|B^X\right\rangle &= \left|B^X_{\mathrm{N}}\right\rangle \otimes \left|B^X_{\mathrm{D}}\right\rangle \\ \left|B^X_{\mathrm{N}}\right\rangle &= \frac{\sqrt{V_{\mathrm{N}}}}{(8\pi^2\alpha')^{\frac{p+1}{4}}} \prod_{\mu\in\mathrm{N}} \left(e^{-\sum_{n=1}^\infty \frac{1}{n}\alpha^\mu_{-n}\tilde{\alpha}_{-n\mu}} \delta_{\mathtt{n}^\mu,0} \sum_{\mathtt{w}'^\mu\in\mathbb{Z}} \delta_{\mathtt{w}^\mu,\mathtt{w}'^\mu} \right) |0\rangle \ , \\ \left|B^X_{\mathrm{D}}\right\rangle &= \frac{(2\pi^2\alpha')^{\frac{26-(p+1)}{4}}}{\sqrt{V_{\mathrm{D}}}} \prod_{i\in\mathrm{D}} \left(e^{\sum_{n=1}^\infty \frac{1}{n}\alpha^i_{-n}\tilde{\alpha}^i_{-n}} \sum_{\mathtt{n}'^i\in\mathbb{Z}} e^{-i\frac{\mathtt{n}'^i}{R^i}x^i} \delta_{\mathtt{n}^i,\mathtt{n}'^i}\delta_{\mathtt{w}^i,0} \right) |0\rangle \ , \end{aligned} \tag{4.8}$$

where V_{N} and V_{D} are respectively the volumes of the Neumann and the Dirichlet directions, i.e. $V_{\mathrm{N}} = \prod_{\mu\in\mathrm{N}} (2\pi R^\mu)$ and $V_{\mathrm{D}} = \prod_{i\in\mathrm{D}} (2\pi R^i)$. We notice that the zero-mode part of the state $\left|B^X_{\mathrm{N}}\right\rangle$ is $\langle \mathtt{n}^\mu \left| \mathtt{n}^\mu = 0\right\rangle \otimes \langle \mathtt{w}^\mu \left| y^\mu = 0\right\rangle$ and that of the state $\left|B^X_{\mathrm{D}}\right\rangle$ is $\langle \mathtt{n}^i \left| x^i\right\rangle \otimes \langle \mathtt{w}^i \left| \mathtt{w}^i = 0\right\rangle$, modulo normalization constants. In the following, we restrict ourselves to the case in which Dp-branes are located at $x^i = 0$.

Let us turn to the ghost sector. We require that the Dirichlet boundary condition should be satisfied by the ghost fields $C(\tau,\sigma)$ and $\bar{C}(\tau,\sigma)$ at $\tau = 0$ on the boundary state:

$$C(0,\sigma)\left|B^{\mathrm{gh}}\right\rangle = 0 \ , \quad \bar{C}(0,\sigma)\left|B^{\mathrm{gh}}\right\rangle = 0 \ . \tag{4.9}$$

In terms of the oscillation modes, these conditions read

$$\begin{gathered} C_0\left|B^{\mathrm{gh}}\right\rangle = 0 \ , \qquad \bar{C}_0\left|B^{\mathrm{gh}}\right\rangle = 0 \ , \\ (\gamma_n - \tilde{\gamma}_{-n})\left|B^{\mathrm{gh}}\right\rangle = 0 \ , \ (\bar{\gamma}_n - \tilde{\bar{\gamma}}_{-n})\left|B^{\mathrm{gh}}\right\rangle = 0 \end{gathered} \tag{4.10}$$

for $\forall n \in \mathbb{Z}$.[5] They coincide with the usual boundary conditions for the b, c ghosts assuming eq. (3.28). This implies that the boundary state $\left|B^{\mathrm{gh}}\right\rangle$ is proportional to the state

$$|B^{\mathrm{gh}}_0\rangle = e^{\sum_{n=1}^\infty \frac{i}{n}\left(\gamma_{-n}\tilde{\bar{\gamma}}_{-n} + \tilde{\gamma}_{-n}\bar{\gamma}_{-n}\right)}|0\rangle \ . \tag{4.11}$$

Let us define $|B_0\rangle$ as

$$|B_0\rangle = \mathcal{N}\left|B^X\right\rangle \otimes |B^{\mathrm{gh}}_0\rangle \ , \tag{4.12}$$

where $\mathcal{N}$ is an arbitrary normalization constant. In what follows, we refer to this state as a boundary state. Since the string field should have α dependence, we need an α dependent version of the boundary state. Let us define $|B_0(l)\rangle$ as

$$|B_0(l)\rangle = |B_0\rangle\, \delta(\alpha - l) \ , \tag{4.13}$$

where the parameter l is an eigenvalue of α, i.e. $\alpha\,|B_0(l)\rangle = l\,|B_0(l)\rangle$.

[5] While the $n = 0$ case of eq. (4.10) is not derived from eq. (4.9), it holds automatically by definition of $(\gamma_0, \bar{\gamma}_0)$ and $(\tilde{\gamma}_0, \tilde{\bar{\gamma}}_0)$.

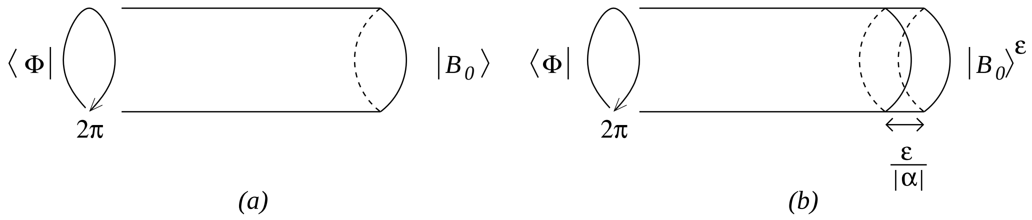

Figure 1: (a) The inner product $\langle \Phi \,| B_0 \rangle$. (b) The inner product $\langle \Phi \,| B_0 \rangle^{\epsilon}$.

4.2 Regularization

The boundary state (4.12) is not normalizable. We need therefore regularize the divergence of the norm of this state, in order to treat it as a string field in string field theory. For this purpose we introduce a regularized boundary state $|B_0\rangle^{\epsilon}$ by attaching a stub to the state $|B_0\rangle$ as depicted in figure 1:

$$|B_0\rangle^{\epsilon} = e^{-\frac{\epsilon}{|\alpha|}\left(L_0+\tilde{L}_0-2\right)}\,|B_0\rangle \ . \tag{4.14}$$

A similar regularization is necessary even for on-shell physical states[11]. This is a BRST invariant regularization[6] because $e^{-\frac{\epsilon}{|\alpha|}\left(L_0+\tilde{L}_0-2\right)}$ commutes with the BRST charge Q_{B}, which can be seen from

$$\{Q_{\mathrm{B}}\,,\, 2\epsilon\bar{\pi}_0\} = \frac{\epsilon}{\alpha}\left(L_0+\tilde{L}_0-2\right) \ . \tag{4.15}$$

A subtlety seems to occur at $\alpha = 0$, because α appears in the form of the absolute value. As is usual in a light-cone formalism, we will exclude the modes with $|\alpha| < \delta$ with some small δ from the spectrum. As we will see, this corresponds to an infrared cut-off on the worldsheet. We will study the theory perturbatively. Therefore we will keep the most dominant contributions in the limit $\epsilon \to 0$ at each order in g, in the following.

We regularize the state (4.13) accordingly:

$$|B_0(l)\rangle^{\epsilon} = |B_0\rangle^{\epsilon}\,\delta(\alpha - l) \ . \tag{4.16}$$

Let us consider the inner product

$$\int dr\ {}^{\epsilon}_{r}\!\left\langle B_0(l)\,\middle|\,B_0(l')\right\rangle^{\epsilon}_{r} = \int drds\,\langle R(s,r)\,|B_0(l)\rangle^{\epsilon}_{s}\,|B_0(l')\rangle^{\epsilon}_{r} \ , \tag{4.17}$$

where dr and ds denote the integration measures for the zero modes defined in eq. (3.7). Eq. (4.17) becomes the cylinder amplitude of a closed string with a fixed circumference 2π propagating through a very short proper time $\frac{2\epsilon}{|l|}$. After the modular transformation for

[6] For BRST invariance, one may also use $\epsilon\left(L_0+\tilde{L}_0-2i\pi_0\bar{\pi}_0-2\right)$ instead of $\frac{\epsilon}{\alpha}\left(L_0+\tilde{L}_0-2\right)$ because

$$\{Q_{\mathrm{B}}\,,\,2\epsilon\alpha\bar{\pi}_0\} = \epsilon\left(L_0+\tilde{L}_0-2i\pi_0\bar{\pi}_0-2\right) \ .$$

This regularization however does not work for our purpose. See the comment at the end of this subsection.

the non-zero mode part and the Poisson resummation for the zero-mode part, we obtain

$$\int dr \, {}^{\epsilon}_{r}\big\langle B_0(l)\,\big|B_0(l')\big\rangle^{\epsilon}_{r} = \mathcal{N}^2\, l'\delta(l+l')\, e^{\frac{\pi^2|l|}{\epsilon}} \frac{1}{\left[\prod_{m=1}^{\infty}\left(1-e^{-\frac{\pi^2|l|}{\epsilon}m}\right)\right]^{24}}$$

$$\times \prod_{\mu\in \mathrm{N}}\left[\sum_{\mathtt{w}^{\mu}\in\mathbb{Z}} e^{-\frac{|l|}{\epsilon}\frac{\pi^2\alpha'}{(R^{\mu})^2}\mathtt{w}^{\mu 2}}\right]\times\prod_{i\in \mathrm{D}}\left[\sum_{\mathtt{n}^{i}\in\mathbb{Z}} e^{-\frac{|l|}{\epsilon}\frac{(\pi R^{i})^2}{\alpha'}\mathtt{n}^{i2}}\right] . \qquad (4.18)$$

The dominant contribution $e^{\frac{\pi^2|l|}{\epsilon}}$ in the limit $\epsilon\to 0$ originates from the propagation of the open string tachyon in the dual channel. We introduce the state $|n(l)\rangle$ defined as

$$|n(l)\rangle = |B_0(l)\rangle^{\epsilon}\, e^{-\frac{\pi^2}{2\epsilon}|l|} . \qquad (4.19)$$

From eq. (4.18), we find that

$$\int dr_r\big\langle n(l)|n(l')\big\rangle_r = \left(\mathcal{N}^2 + \mathcal{O}\left(e^{-\frac{1}{\epsilon}}\right)\right) l'\,\delta(l+l') . \qquad (4.20)$$

Thus the state $|n(l)\rangle$ is normalizable.

A comment is in order. Naively speaking

$$\int dr \, {}_{r}\big\langle B_0(l)\,\big|B_0(l')\big\rangle_r = 0, \qquad (4.21)$$

because the wave function for $|B_0(l)\rangle$ lacks factors of π_0 and $\bar{\pi}_0$. However in eq. (4.17), $e^{-\frac{\epsilon}{|\alpha|}(L_0+\tilde{L}_0-2)}$ provides these and we get a nonvanishing answer for the inner product.

4.3 An expansion of the string field

Now let us define the creation and annihilation operators of the closed strings whose wave functions are proportional to the boundary states. The states $\{|n(l)\rangle\,,|n(-l)\rangle\}$ with $l>0$ are normalizable as stated above and the inner products (4.20) among them are non-degenerate. This enables us to choose a complete basis of the Hilbert space which consists of these states and their orthogonal complement. Taking also the states $\bar{\pi}_0\,|n(l)\rangle$ into account, we can expand $|\Phi\rangle$ as

$$|\Phi\rangle = \int_0^{\infty} dl\Big[\,|n(l)\rangle\,\phi(l) + \bar{\pi}_0\,|n(l)\rangle\,\chi(l) + |n(-l)\rangle\,\bar{\phi}(l) + \bar{\pi}_0\,|n(-l)\rangle\,\bar{\chi}(l) + \cdots\Big] , \qquad (4.22)$$

where '$\cdots$' denotes the contributions from the other states. The wave functions $\phi(l)$, $\bar{\phi}(l)$, $\chi(l)$, $\bar{\chi}(l)$ etc. are the fields to be quantized in the second quantization.[7] $\phi(l)$ and $\bar{\phi}(l)$ can be considered as the annihilation and creation operators for the closed strings corresponding to the boundary state. Let us divide $|\Phi\rangle$ into the creation and annihilation parts as follows:

$$\begin{aligned}
|\Phi\rangle &= |\psi\rangle + |\bar{\psi}\rangle \ , \\
|\psi\rangle &= \int_0^{\infty} dl\Big[\,|n(l)\rangle\,\phi(l) + \bar{\pi}_0\,|n(l)\rangle\,\chi(l) + \cdots\Big] , \\
|\bar{\psi}\rangle &= \int_0^{\infty} dl\Big[\,|n(-l)\rangle\,\bar{\phi}(l) + \bar{\pi}_0\,|n(-l)\rangle\,\bar{\chi}(l) + \cdots\Big] .
\end{aligned} \qquad (4.23)$$

[7]The wave functions $\phi(l)$ etc. depend on the proper time t: $\phi(t,l)$. We suppress the proper time in the arguments for simplicity.

The string field $|\Phi\rangle$ satisfies the reality condition:

$$\langle\Phi_{\rm hc}| = \langle\Phi| \ , \tag{4.24}$$

where $\langle\Phi_{\rm hc}| \equiv (|\Phi\rangle)^\dagger$ denotes the hermitian conjugate of $|\Phi\rangle$, and $\langle\Phi|$ denotes the BPZ conjugate of $|\Phi\rangle$ defined in eq. (3.16), respectively. Since the BPZ conjugation flips the sign of the string length α, the reality condition (4.24) implies that

$$\langle\psi_{\rm hc}| = \langle\bar{\psi}| \ , \quad \langle\bar{\psi}_{\rm hc}| = \langle\psi| \ . \tag{4.25}$$

Combined with the relation

$$\langle n(l)_{\rm hc}| = \langle n(-l)| \ , \tag{4.26}$$

eq. (4.25) leads to

$$\phi^\dagger(l) = \bar{\phi}(l) \ , \quad \chi^\dagger(l) = \bar{\chi}(l) \ . \tag{4.27}$$

The BRST transformations $\delta_{\rm B}\phi(l)$ and $\delta_{\rm B}\bar{\phi}(l)$ for the component fields $\phi(l)$ and $\bar{\phi}(l)$ can be calculated from eq. (3.26). Considering the idempotency equations [12] satisfied by the boundary states, we expect that the nonlinear terms in the transformation takes a very simple form. Indeed we obtain

$$\begin{aligned}
\frac{4C\epsilon^3}{g\mathcal{N}}\delta_{\rm B}\phi(l) &= \frac{C}{g\mathcal{N}}\epsilon^2\left(\frac{\partial}{\partial l}+\frac{\pi^2}{2\epsilon}\right)(l\chi(l)) - \int_0^l dl_1\, l_1(l-l_1)\chi(l_1)\phi(l-l_1) \\
&\quad - \int_0^\infty dl_1\, l_1(l+l_1)\left[\chi(l+l_1)\bar{\phi}(l_1)+\bar{\chi}(l_1)\phi(l+l_1)\right] \\
&\quad + \cdots \ , \\
\frac{4C\epsilon^3}{g\mathcal{N}}\delta_{\rm B}\bar{\phi}(l) &= -\frac{C}{g\mathcal{N}}\epsilon^2\left(\frac{\partial}{\partial l}+\frac{\pi^2}{2\epsilon}\right)(l\bar{\chi}(l)) + \int_0^l dl_1\, l_1(l-l_1)\bar{\chi}(l_1)\bar{\phi}(l-l_1) \\
&\quad + \int_0^\infty dl_1\, l_1(l+l_1)\left[\bar{\chi}(l+l_1)\phi(l_1)+\chi(l_1)\bar{\phi}(l+l_1)\right] \\
&\quad + \cdots \ ,
\end{aligned} \tag{4.28}$$

where

$$C = \frac{1}{4\pi^3}\frac{(2\pi^2\alpha')^{\frac{13}{2}}}{(4\pi^2\alpha')^{\frac{p+1}{2}}}\sqrt{\frac{V_{\rm N}}{V_{\rm D}}} \ . \tag{4.29}$$

The derivation of eq. (4.28) is presented in Appendix A. As is intuitively clear, a boundary state split into two makes two boundary states and two boundary states joined together makes a boundary state. Such contributions are written explicitly in eq. (4.28). However, a boundary state joined to a different state makes a state different from the boundary state. Such contributions are denoted by '$\cdots$'. Notice that each term in '$\cdots$' should be a product of one annihilation operator other than $\phi(l), \chi(l)$ and one creation operator other than $\bar{\phi}(l), \bar{\chi}(l)$.

5. Solitonic operators

In this section, we will construct solitonic operators made from the ϕ and $\bar{\phi}$ and study their properties.

5.1 Canonical quantization

Let us canonically quantize the string fields defined in eq. (4.22) first. The kinetic term of the action (3.24) can be written as

$$\begin{aligned} S_K &= \frac{1}{2}\int dt \int d1d2\, \langle R(1,2)\,|\Phi\rangle_1 \left(i\frac{\partial}{\partial t} - \frac{L_0^{(2)}+\tilde{L}_0^{(2)}-2}{\alpha_2}\right)|\Phi\rangle_2 \\ &= \int dt \int d1d2\, \langle R(1,2)\,|\bar{\psi}\rangle_1 \left(i\frac{\partial}{\partial t} - \frac{L_0^{(2)}+\tilde{L}_0^{(2)}-2}{\alpha_2}\right)|\psi\rangle_2 \\ &= \int dt \int d2\, {}_2\langle\bar{\psi}| \left(i\frac{\partial}{\partial t} - \frac{L_0^{(2)}+\tilde{L}_0^{(2)}-2}{\alpha_2}\right)|\psi\rangle_2 \ . \end{aligned} \tag{5.1}$$

In the same way as was performed in the light-cone string field theory [10, 16], we obtain the canonical commutation relation

$$\left[|\psi\rangle_r\,, {}_s\langle\bar{\psi}|\right] = I(r,s) \qquad \Leftrightarrow \qquad \left[|\psi\rangle_r\,, |\bar{\psi}\rangle_s\right] = |R(r,s)\rangle \ , \tag{5.2}$$

where $I(r,s)$ is defined as

$$I(r,s) = \int du\, \langle R(u,s)\,|R(r,u)\rangle \ . \tag{5.3}$$

$I(r,s)$ serves as the identity operator. In fact, the following relations hold for an arbitrary string field $|\Psi\rangle$,

$$\int ds\, I(r,s)\,|\Psi\rangle_s = |\Psi\rangle_r \ , \qquad \int dr\, {}_r\langle\Psi|\, I(r,s) = {}_s\langle\Psi| \ . \tag{5.4}$$

Multiplying the second equation in eq. (5.2) by $\int dr\, {}_r\langle n(-l_r)| \int ds\, {}_s\langle n(l_s)|$ from the left, we have

$$\left[\phi(l_r), \bar{\phi}(l_s)\right] = \frac{1}{\mathcal{N}^2 l_r}\delta(l_r - l_s) \ . \tag{5.5}$$

One can also derive this commutation relation directly from the action (5.1) expressed in terms of the component fields:

$$S_K = \mathcal{N}^2 \int dt \int_0^\infty dl\, l\, \bar{\phi}(l) i\frac{\partial\phi(l)}{\partial t} + \cdots \ . \tag{5.6}$$

The vacuum state $|0\rangle\rangle$ in the second quantization is defined by

$$|\psi\rangle|0\rangle\rangle = 0 \ , \quad \langle\langle 0|\langle\bar{\psi}| = 0 \ . \tag{5.7}$$

This yields

$$\phi(l)|0\rangle\rangle = \chi(l)|0\rangle\rangle = 0 \ , \quad \langle\langle 0|\bar{\phi}(l) = \langle\langle 0|\bar{\chi}(l) = 0, \qquad \text{for } l > 0 \ . \tag{5.8}$$

We take the normalization of the vacuum state $|0\rangle\rangle$ as $\langle\langle 0|0\rangle\rangle = 1$.

5.2 Solitonic states

The right hand sides of eq. (4.28) look quite like those of eq. (2.5). Indeed if we replace

$$\left[\frac{1}{g_s^2}\int_0^\infty dl'l'\epsilon(l')T(l')\,,\ \cdot\ \right] \ \rightarrow\ \frac{4C\epsilon^3}{g\mathcal{N}}\delta_{\rm B}\left(\ \cdot\ \right)\ ,$$

$$\frac{\sqrt{2}}{g_s\mathcal{N}l}\psi(l)\rightarrow\phi(l)\ ,\quad \frac{g_s}{\sqrt{2}\mathcal{N}}\psi^\dagger(l)\rightarrow\bar{\phi}(l)\ ,\quad \epsilon(l)\rightarrow\bar{\chi}(l)\ , \tag{5.9}$$

in eq. (2.5), we get exactly the nonlinear terms involving $\bar{\chi}(l)$ on the right hand sides of eq. (4.28). Moreover the commutation relation (2.1) becomes eq. (5.5) by such replacements. Therefore we expect that operators in the following form can be used to construct BRST invariant operators:

$$\exp\left[\pm\sqrt{2}\mathcal{N}\int_0^\infty dl\ e^{-\zeta l}\bar{\phi}(l)\right]\,\exp\left[\mp\sqrt{2}\mathcal{N}\int_0^\infty dl'\ e^{\zeta l'}\phi(l')\right]. \tag{5.10}$$

Taking the linear terms on the right hand sides of eq. (4.28) into account, we define

$$\mathcal{V}(\zeta)=\lambda\exp\left[\pm\sqrt{2}\mathcal{N}\int_0^\infty dl\ e^{-\zeta l}\bar{\phi}(l)\right]\,\exp\left[\mp\sqrt{2}\mathcal{N}\int_0^\infty dl'\ e^{\zeta l'}\phi(l')\right]\,e^{\pm\frac{C\epsilon^2}{\sqrt{2}g}\left(\zeta+\frac{\pi^2}{2\epsilon}\right)^2} \tag{5.11}$$

where λ is a constant. Actually we cannot make BRST invariant operators from $\mathcal{V}(\zeta)$. Rather we will show that we can construct BRST invariant states by acting $\int d\zeta\mathcal{V}(\zeta)$ on the vacuum $|0\rangle\rangle$.

As a warm-up, let us show that

$$\begin{aligned} |D\rangle\rangle &\equiv \int d\zeta\,\mathcal{V}(\zeta)|0\rangle\rangle \\ &= \lambda\int d\zeta\exp\left[\pm\sqrt{2}\mathcal{N}\int_0^\infty dl\,e^{-\zeta l}\bar{\phi}(l)\pm\frac{C\epsilon^2}{\sqrt{2}g}\left(\zeta+\frac{\pi^2}{2\epsilon}\right)^2\right]|0\rangle\rangle \end{aligned} \tag{5.12}$$

is BRST invariant:

$$\delta_{\rm B}|D\rangle\rangle=0\ . \tag{5.13}$$

The BRST transformation can be calculated by using eq. (4.28) as

$$\begin{aligned} &\frac{4C\epsilon^3}{g\mathcal{N}}\,\delta_{\rm B}\left(e^{\pm\sqrt{2}\mathcal{N}\int_0^\infty dl\,e^{-\zeta l}\bar{\phi}(l)}\right)|0\rangle\rangle \\ &=\left[\mp\frac{C\epsilon^2\sqrt{2}}{g}\int_0^\infty dl\,e^{-\zeta l}\left(\frac{\partial}{\partial l}+\frac{\pi^2}{2\epsilon}\right)(l\bar{\chi}(l))\right. \\ &\qquad\pm\sqrt{2}\mathcal{N}\int_0^\infty dl\,e^{-\zeta l}\int_0^l dl_1\,l_1(l-l_1)\bar{\chi}(l_1)\bar{\phi}(l-l_1) \\ &\qquad\left.+\int_0^\infty dl\int_0^\infty dl_1\,e^{-\zeta(l+l_1)}(l+l_1)\bar{\chi}(l+l_1)\right]e^{\pm\sqrt{2}\mathcal{N}\int_0^\infty dl\,e^{-\zeta l}\bar{\phi}(l)}\,|0\rangle\rangle\ . \end{aligned} \tag{5.14}$$

Here we have used $\phi(l)|0\rangle\rangle=\chi(l)|0\rangle\rangle=0$ and the fact that '$\cdots$' in eq. (4.28) does not contribute because it includes annihilation operators other than ϕ. It is useful to introduce the Laplace transforms $\tilde{\bar{\phi}}(\zeta)$ and $\tilde{\bar{\chi}}(\zeta)$ of $\bar{\phi}(l)$ and $\bar{\chi}(l)$ defined as

$$\tilde{\bar{\phi}}(\zeta)=\int_0^\infty dl\,e^{-\zeta l}\bar{\phi}(l)\ ,\qquad \tilde{\bar{\chi}}(\zeta)=\int_0^\infty dl\,e^{-\zeta l}\bar{\chi}(l)\ . \tag{5.15}$$

The following identities hold,

$$
\begin{aligned}
\int_0^\infty dl\, e^{-\zeta l}\left(\frac{\partial}{\partial l}+\frac{\pi^2}{2\epsilon}\right)(l\bar{\chi}(l)) &= -\left(\zeta+\frac{\pi^2}{2\epsilon}\right)\frac{\partial}{\partial\zeta}\tilde{\bar{\chi}}(\zeta)\ , \\
\int_0^\infty dl\, e^{-\zeta l}\int_0^l dl_1\, l_1(l-l_1)\bar{\chi}(l_1)\bar{\phi}(l-l_1) &= \frac{\partial\tilde{\bar{\chi}}(\zeta)}{\partial\zeta}\frac{\partial\tilde{\bar{\phi}}(\zeta)}{\partial\zeta}\ , \\
\int_0^\infty dl\, e^{-\zeta l}\int_0^\infty dl_1\, e^{-\zeta l_1}(l+l_1)\bar{\chi}(l+l_1) &= \frac{\partial^2}{\partial\zeta^2}\tilde{\bar{\chi}}(\zeta)\ .
\end{aligned} \tag{5.16}
$$

Combining these relations with eq. (5.14), we obtain

$$
\begin{aligned}
&\frac{4C\epsilon^3}{g\mathcal{N}}\,\delta_B\left(\exp\left[\pm\sqrt{2}\mathcal{N}\int_0^\infty dl\, e^{-\zeta l}\bar{\phi}(l)\pm\frac{C\epsilon^2}{\sqrt{2}g}\left(\zeta+\frac{\pi^2}{2\epsilon}\right)^2\right]\right)|0\rangle\!\rangle \\
&\quad=\frac{\partial}{\partial\zeta}\left(\frac{\partial\tilde{\bar{\chi}}(\zeta)}{\partial\zeta}\exp\left[\pm\sqrt{2}\mathcal{N}\int_0^\infty dl\, e^{-\zeta l}\bar{\phi}(l)\pm\frac{C\epsilon^2}{\sqrt{2}g}\left(\zeta+\frac{\pi^2}{2\epsilon}\right)^2\right]\right)|0\rangle\!\rangle\ .
\end{aligned} \tag{5.17}
$$

Taking eq. (5.12) into account, we find that this equation implies the BRST invariance (5.13) of the state $|D\rangle\!\rangle$.

$|D\rangle\!\rangle$ can be considered as a state in which D-branes are excited. Actually as we will see in the next subsection, two D-branes are there. In order to have more D-branes, we just have to operate $\int d\zeta\mathcal{V}(\zeta)$ successively on the vacuum $|0\rangle\!\rangle$, namely we construct states

$$
\left(\int d\zeta\mathcal{V}(\zeta)\right)^n|0\rangle\!\rangle\ , \tag{5.18}
$$

for $n>0$. They are BRST invariant, because $\int d\zeta\mathcal{V}(\zeta)$ is BRST invariant modulo terms which annihilate the states $(\int d\zeta\mathcal{V}(\zeta))^n|0\rangle\!\rangle$, $n\geq 0$. This can be seen as follows. Under the BRST transformation (4.28), $\mathcal{V}(\zeta)$ transforms as

$$
\frac{4C\epsilon^3}{g\mathcal{N}}\,\delta_{\rm B}\mathcal{V}(\zeta)=\frac{\partial}{\partial\zeta}\left[\left(\frac{\partial\tilde{\bar{\chi}}(\zeta)}{\partial\zeta}-\frac{\partial\tilde{\chi}(\zeta)}{\partial\zeta}\right)\mathcal{V}(\zeta)\right]+\left(\pm\frac{2\sqrt{2}C\epsilon^2}{g}\zeta\frac{\partial\tilde{\chi}(\zeta)}{\partial\zeta}+\cdots\right)\mathcal{V}(\zeta)\ , \tag{5.19}
$$

where '$\cdots$' denotes the terms which includes annihilation operators other than $\phi(l)$, and $\tilde{\phi}(\zeta)$ and $\tilde{\chi}(\zeta)$ are the Laplace transforms of $\phi(l)$ and $\chi(l)$ defined as

$$
\tilde{\phi}(\zeta)=\int_0^\infty dl\, e^{\zeta l}\phi(l)\ ,\quad \tilde{\chi}(\zeta)=\int_0^\infty dl\, e^{\zeta l}\chi(l)\ . \tag{5.20}
$$

We can prove eq. (5.19) with the help of the relation

$$
\begin{aligned}
&\int_0^\infty dl\, e^{-\zeta l}\int_0^\infty dl_1\, l_1(l+l_1)\chi(l_1)\bar{\phi}(l+l_1) \\
&\quad=-\frac{\partial\tilde{\chi}(\zeta)}{\partial\zeta}\frac{\partial\tilde{\bar{\phi}}(\zeta)}{\partial\zeta}-\int_0^\infty dl\int_0^\infty dl_1\, e^{\zeta l}l_1(l+l_1)\chi(l+l_1)\bar{\phi}(l_1)\ .
\end{aligned} \tag{5.21}
$$

The terms in the parenthesis ( ) in the second term on the right hand side of eq. (5.19) commute with $\mathcal{V}(\zeta)$ and annihilate $|0\rangle\!\rangle$. Thus we find that the states of the form $(\int d\zeta\mathcal{V}(\zeta))^n|0\rangle\!\rangle$ are BRST invariant.

5.3 Vacuum amplitude

In order to compare our description of D-branes constructed above with the usual one, let us compute the vacuum amplitude in the presence of D-branes in our formalism. Since $|D\rangle\!\rangle$ is considered as a state with some D-branes excited, the vacuum amplitude in the presence of D-branes can be given as

$$\lim_{T\to\infty} \langle\!\langle D|e^{-iT\hat{H}}|D\rangle\!\rangle \ , \tag{5.22}$$

where $\hat{H}$ is the second-quantized Hamiltonian. Notice that the time variable in the OSp invariant string field theory is not the physical time but with "topological" nature because the Hamiltonian $\hat{H}$ is BRST exact. This can be seen from the expression (3.27). Therefore $\langle\!\langle D|e^{-iT\hat{H}}|D\rangle\!\rangle$ can be transformed into a form which is independent of the value of T. By comparing $\langle\!\langle D|e^{-iT\hat{H}}|D\rangle\!\rangle$ with the usual vacuum amplitude, we can see how many D-branes are there in the state $|D\rangle\!\rangle$.

In order to do so, let us first perform the integration over ζ in eq. (5.12). Perturbatively the factor

$$\exp\left[\pm\frac{C\epsilon^2}{\sqrt{2}g}\left(\zeta+\frac{\pi^2}{2\epsilon}\right)^2\right] \tag{5.23}$$

in the integrand in eq. (5.12) is the most dominant. Therefore, through the saddle point approximation we obtain

$$|D\rangle\!\rangle \simeq \lambda' \exp\left[\pm\sqrt{2}\mathcal{N}\int_0^\infty dl\, e^{\frac{\pi^2}{2\epsilon}l}\bar{\phi}(l)\right]|0\rangle\!\rangle \ , \tag{5.24}$$

where $\lambda' = \sqrt{\mp\frac{\sqrt{2}\pi g}{C\epsilon^2}}\lambda$. One can find that

$$\begin{aligned}\bar{\phi}(l)\, e^{\frac{\pi^2}{2\epsilon}l} &= -\frac{1}{\mathcal{N}^2 l}\int dr\, e^{\frac{\pi^2}{2\epsilon}l}\, {}_r\langle n(l)|\bar{\psi}\rangle_r \\ &= -\frac{1}{\mathcal{N}^2}\int dr\,\frac{1}{l}\, {}^{\epsilon}_{r}\langle B_0(l)|\bar{\psi}\rangle_r = \frac{1}{\mathcal{N}^2}\int dr\,\frac{1}{l}\, {}_r\langle\bar{\psi}|B_0(l)\rangle^{\epsilon}_r \ .\end{aligned} \tag{5.25}$$

Plugging this relation into eq. (5.24), we obtain the expression of the state $|D\rangle\!\rangle$ in terms of the string state $|\psi\rangle$ and $|B_0(l)\rangle^{\epsilon}$ as follows:

$$|D\rangle\!\rangle = \lambda' \exp\left[\pm\int dr\int_0^\infty \frac{dl}{l}\,\frac{\sqrt{2}}{\mathcal{N}}\, {}_r\langle\bar{\psi}|B_0(l)\rangle^{\epsilon}_r\right]|0\rangle\!\rangle \ . \tag{5.26}$$

Note that the divergent factor $e^{\frac{\pi^2}{2\epsilon}l}$ ($l>0$) is miraculously canceled by the regularization factor $e^{-\frac{\pi^2}{2\epsilon}|l|}$ in $|n(l)\rangle$ and we can express $|D\rangle\!\rangle$ in terms of $|B_0(l)\rangle$.

Now let us evaluate $\langle\!\langle D|e^{-iT\hat{H}}|D\rangle\!\rangle$. Perturbatively the lowest order contribution can be obtained by replacing the Hamiltonian $\hat{H}$ with its free part $\hat{H}_0$. Substituting

$$\begin{aligned}\langle\!\langle D| &= \lambda'^{\dagger}\langle\!\langle 0|\exp\left[\pm\int dr\int_0^\infty\frac{dl}{l}\frac{\sqrt{2}}{\mathcal{N}}\, {}^{\epsilon}_{r}\langle B_0(-l)|\psi\rangle_r\right] \ , \\ \hat{H}_0 &= \int dr\, {}_r\langle\bar{\psi}|\,\frac{L_0^{(r)}+\tilde{L}_0^{(r)}-2}{\alpha_r}\,|\psi\rangle_r \ ,\end{aligned} \tag{5.27}$$

into the expression and using the commutation relation (5.2), we find that

$$\begin{aligned}
&\langle\!\langle D|e^{-iT\hat{H}_0}|D\rangle\!\rangle \\
&= |\lambda'|^2 \langle\!\langle 0|\exp\left[\frac{2}{\mathcal{N}^2}\int dr\int_0^\infty \frac{dl}{l}\, {}^{\epsilon}_{r}\langle B_0(-l)|\psi\rangle_r\; e^{-iT\hat{H}_0}\int dr'\int_0^\infty \frac{dl'}{l'}\, {}_{r'}\langle\bar{\psi}|B_0(l')\rangle^{\epsilon}_{r'}\right]|0\rangle\!\rangle \\
&= |\lambda'|^2 \exp\left[\frac{2}{\mathcal{N}^2}\int_0^\infty \frac{dl}{l}\int_0^\infty \frac{dl'}{l'}\int dr\, {}^{\epsilon}_{r}\langle B_0(-l')|\, e^{-iT\frac{L_0^{(r)}+\tilde{L}_0^{(r)}-2}{\alpha_r}}\,|B_0(l)\rangle^{\epsilon}_{r}\right]. \qquad (5.28)
\end{aligned}$$

Here we have used the relation

$$\langle\!\langle 0|\, {}^{\epsilon}_{r}\langle B_0(-l)\,|\,\psi\rangle_r\; \hat{H}_0 = \langle\!\langle 0|\, {}^{\epsilon}_{r}\langle B_0(-l)|\,\frac{L_0^{(r)}+\tilde{L}_0^{(r)}-2}{\alpha_r}\,|\psi\rangle_r\ , \qquad (5.29)$$

which follows from eq. (5.2).

The quantity in the exponent of eq. (5.28) should be compared with the cylinder amplitude in the usual formulation. After the integrations over α, π_0, $\bar{\pi}_0$ and l', we obtain the integration measure for l as

$$\int_0^\infty dl\frac{T}{l^2} = \int_0^\infty d\left(\frac{T}{l}\right)\ . \qquad (5.30)$$

Since T is fixed, this is exactly the integration over the parameter in front of $L_0+\tilde{L}_0-2$. Therefore the integration over l is transformed into the one for the moduli parameter of the cylinder and the result is in a form which is independent of the value of T. The overlap between the boundary states in the exponent on the right hand side of eq. (5.28) can readily be obtained from eq. (4.18) by replacing ϵ in eq. (4.18) by $iT/2$. Introducing $\tau' \equiv -\pi\frac{l}{T}$, we obtain

$$\begin{aligned}
&\langle\!\langle D|e^{-iT\hat{H}_0}|D\rangle\!\rangle \\
&= |\lambda'|^2\exp\left[4\int_0^\infty \frac{d\tau'}{2\tau'}\eta(-\tau')^{-24}\prod_{\mu\in \mathrm{N}}\left(\sum_{m\in\mathbb{Z}} e^{-i\frac{2\pi\alpha'}{(R^\mu)^2}\tau' m^2}\right)\prod_{i\in \mathrm{D}}\left(\sum_{n\in\mathbb{Z}} e^{-i\frac{2\pi(R^i)^2}{\alpha'}\tau' n^2}\right)\right] \qquad (5.31)
\end{aligned}$$

Notice that the arbitrary normalization constant $\mathcal{N}$ does not appear in this final answer. We can see that the exponent of eq. (5.31) reproduces four times the annulus amplitude for one D-brane. This result implies that $|D\rangle\!\rangle$ is the state in which two D-branes or ghost D-branes are excited. Since $\bar{\phi}$ generates boundaries on the worldsheet, depending on which sign in eq. (5.11) is chosen, D-branes or ghost D-branes are excited. Similar calculations yield that $\int d\zeta\mathcal{V}(\zeta)$ creates two D-branes or ghost D-branes.

In this subsection, we have shown that the cylinder amplitudes for D-branes are reproduced in our formulation. What is remarkable is that the integration measure for the moduli of the cylinder appear from the integration over the length of the string. It would be an intriguing problem to check if the integration measures for the higher genus graphs appear in a similar way.

6. Discussions

In this paper, we have constructed solitonic operators which create D-branes. Although we started from the nonnormalizable state (4.12), the divergent factors cancel with each other and the cylinder amplitude is reproduced. The cancellation occurred because of the factor $\exp\left[\pm\frac{C\epsilon^2}{\sqrt{2}g}\left(\zeta+\frac{\pi^2}{2\epsilon}\right)^2\right]$ in $\mathcal{V}(\zeta)$. It originates from the term $-i\pi_0\partial_\alpha$ in the BRST charge (3.8). This term is peculiar to the OSp invariant theory and it seems that our construction works only for this theory. The exponent of the exponential factor we are discussing may be interpreted as the potential for the open string tachyon. Indeed ζ appears in the form of $\exp(-\zeta l)$ in front of $\phi(l)$ and can be considered as a constant tachyon background. It will be useful to interpret various quantities in our construction in terms of open string language using for example the methods in ref. [25].

In our construction, we do not describe D-branes as solutions of equations of motion. Rather we construct the solitonic operator $\int d\zeta\mathcal{V}(\zeta)$, where the form of $\mathcal{V}(\zeta)$ looks quite like the bosonization formula. Another way to look at our results is as follows. We may write

$$\int_0^\infty \frac{dl}{l}\,{}^{\epsilon}_{r}\langle B_0(l)\,|\bar{\psi}\rangle_r = \int_{-\infty}^\infty \frac{dl}{l}\,{}^{\epsilon}_{r}\langle B_0(l)\,|\bar{\psi}\rangle_r = {}^{\epsilon}_{r}\langle B\,|\bar{\psi}\rangle_r \ , \tag{6.1}$$

by introducing the state $|B\rangle^\epsilon$ defined as

$$|B\rangle^\epsilon = |B_0\rangle^\epsilon\,\frac{1}{\alpha} \ . \tag{6.2}$$

This yields

$$|D\rangle\!\rangle = \lambda'\exp\left[\mp\frac{\sqrt{2}}{\mathcal{N}}\int dr\,{}^{\epsilon}_{r}\langle B\,|\bar{\psi}\rangle_r\right]\,|0\rangle\!\rangle \ . \tag{6.3}$$

The state $|B\rangle^\epsilon$ is regarded as a regularized version of the state $|B\rangle \equiv |B_0\rangle\,\frac{1}{\alpha}$. We find that the states $|B\rangle$ and $|B\rangle^\epsilon$ are annihilated by the BRST charge $Q_{\rm B}$:

$$Q_{\rm B}\,|B\rangle = Q_{\rm B}\,|B\rangle^\epsilon = 0 \ . \tag{6.4}$$

We can extend eq. (6.3) by including the dependence on the annihilation modes $|\psi\rangle$ into

$$|D\rangle\!\rangle = \lambda' \ :\exp\left[\mp\frac{\sqrt{2}}{\mathcal{N}}\int dr\,{}^{\epsilon}_{r}\langle B|\,\Phi\rangle_r\right]:|0\rangle\!\rangle \ , \tag{6.5}$$

where : : means the normal ordering in which the annihilation mode $|\psi\rangle$ should be moved to the right of the creation mode $|\bar{\psi}\rangle$. Taking account of the relation (6.4), we notice that the exponent of eq. (6.5) takes a form quite similar to the interaction term of a closed string with a D-brane introduced by ref. [2] into the action of the HIKKO closed string field theory [26]. Therefore we can say that D-branes are introduced as a source of closed strings.

As we mentioned, we should introduce a cut-off δ so that $|\alpha|, |l| > \delta$. Since l works as a moduli, this cut-off will remove the singularity occurring in the limit where a part of the worldsheet becomes a very long cylinder. Therefore every equation in this paper should be understood with such a cut-off being introduced. In the first relation in eq. (5.16), we have ignored the contribution from the boundary term $e^{-\zeta l} l \bar{\chi}(l)\Big|_{l=0}$. Due to the cut-off, we should take care of the contribution of the boundary term near $l = 0$, which violates the BRST invariance of the state $|D\rangle\rangle$. We should therefore employ the Fischler-Susskind mechanism [27], in order to remedy the violation of the BRST invariance. This modifies the equations of motion for light states of closed string. This is also consistent with the picture of the D-brane as a source of closed strings. Our treatment in section 5 is perturbative and ignores such higher order effects.

There are several problems that remain to be studied. One immediate question is why our solitonic operators create two D-branes or ghost D-branes. We are not sure if there exist operators which create one D-brane or ghost D-brane. It will be an intriguing problem to look for such operators. In this paper, we have not fixed the sign in the exponents of eqs. (5.11) and (5.12). From the calculation (5.31), one can find that if the state $|D\rangle\rangle$ with one sign is identified with ordinary D-branes, the state with the other sign should be identified with ghost D-branes [14]. To determine which sign corresponds to which will be another interesting problem. Of course, our results should be generalized to the superstring case. In order to do so, we should first construct the OSp invariant theory for superstrings.

Acknowledgments

We would like to thank K. Araki, N. Hatano, S. Katagiri and T. Saitou for discussions. This work is supported in part by Grants-in-Aid for Scientific Research 13135224.

A. Derivation of eq. (4.28)

In this appendix, we provide some details of the calculation to get eq. (4.28) from the BRST transformation (3.26) for the string field $|\Phi\rangle$ given in eq. (4.22). For later convenience, we introduce the notation

$$\vec{\phi}(l) = \begin{cases} \phi(l) & \text{for } l > 0 \\ \bar{\phi}(-l) & \text{for } l < 0 \end{cases} , \quad \vec{\chi}(l) = \begin{cases} \chi(l) & \text{for } l > 0 \\ \bar{\chi}(-l) & \text{for } l < 0 \end{cases} . \tag{A.1}$$

In order to get $\delta_{\mathrm{B}}\phi$ and $\delta_{\mathrm{B}}\bar{\phi}$, we consider the inner product of $\langle n(-l)|$ and eq. (3.26). As for the left hand side of the BRST transformation, one can readily find that

$$n(-l) \cdot \delta_{\mathrm{B}}\Phi \equiv \int dr\, {}_r\langle n(-l)\,|\delta_{\mathrm{B}}\Phi\rangle_r = \mathcal{N}^2 l\, \delta_{\mathrm{B}}\vec{\phi}(l) = \begin{cases} \mathcal{N}^2 l \delta_{\mathrm{B}}\phi(l) & \text{for } l > 0 \\ \mathcal{N}^2 l \delta_{\mathrm{B}}\bar{\phi}(-l) & \text{for } l < 0 \end{cases} . \tag{A.2}$$

Let us turn to the right hand side of the BRST transformation. First, we consider the linear term $Q_{\mathrm{B}}|\Phi\rangle$. Using eq. (3.15), we find that

$$\begin{aligned} n(-l) \cdot Q_{\mathrm{B}}\Phi &\equiv \int d1d2 \langle R(1,2)|n(-l)\rangle_1 Q_{\mathrm{B}}^{(2)}|\Phi\rangle_2 \\ &= -\int d1d2 \langle R(1,2)|Q_{\mathrm{B}}^{(1)}|n(-l)\rangle_1 |\Phi\rangle_2 \,. \end{aligned} \tag{A.3}$$

The state $|n(-l)\rangle$ is expressed as

$$|n(-l)\rangle = |B\rangle^\epsilon\, e^{-\frac{\pi^2}{2\epsilon}|l|}\alpha\delta(\alpha+l)\ , \tag{A.4}$$

where $|B\rangle^\epsilon$ is the state introduced in eq. (6.2). Combined with eq. (6.4), this leads to

$$\begin{aligned} Q_{\rm B}|n(-l)\rangle &= Q_{\rm B}\left(|B\rangle^\epsilon\, e^{-\frac{\pi^2}{2\epsilon}|l|}\alpha\delta(\alpha+l)\right) = |B\rangle^\epsilon\, e^{-\frac{\pi^2}{2\epsilon}|l|}Q_{\rm B}\left(\alpha\delta(\alpha+l)\right) \\ &= |B\rangle^\epsilon\, e^{-\frac{\pi^2}{2\epsilon}|l|}(-i\pi_0)\frac{\partial}{\partial\alpha}\left(\alpha\delta(\alpha+l)\right) = |B\rangle^\epsilon\, e^{-\frac{\pi^2}{2\epsilon}|l|}i\pi_0 l\frac{\partial}{\partial l}\delta(\alpha+l)\ . \end{aligned} \tag{A.5}$$

Substituting this equation into eq. (A.3), we have

$$\begin{aligned} n(-l)\cdot Q_B\Phi &= -ie^{-\frac{\pi^2}{2\epsilon}|l|}l\int d1d2\,\langle R(1,2)|\pi_0^{(1)}\frac{\partial}{\partial l}\delta(\alpha_1+l)\,|B\rangle_1^\epsilon\,|\Phi\rangle_2 \\ &= ie^{-\frac{\pi^2}{2\epsilon}|l|}l\int d1d2\,\langle R(1,2)|\frac{\partial}{\partial l}\delta(\alpha_1+l)\,|B\rangle_1^\epsilon\,\pi_0^{(2)}\,|\Phi\rangle_2 \\ &= ie^{-\frac{\pi^2}{2\epsilon}|l|}l\int d1d2\,\frac{\partial}{\partial l}\left(\langle R(1,2)|\,\delta(\alpha_1+l)\,|B\rangle_1^\epsilon\,\pi_0^{(2)}\,|\Phi\rangle_2\right) \\ &= ie^{-\frac{\pi^2}{2\epsilon}|l|}l\int d1d2\,\frac{\partial}{\partial l}\left(\langle R(1,2)|\,\frac{-1}{l}e^{\frac{\pi^2}{2\epsilon}|l|}\,|n(-l)\rangle_1\,\pi_0^{(2)}\,|\Phi\rangle_2\right) \\ &= -ie^{-\frac{\pi^2}{2\epsilon}|l|}l\frac{\partial}{\partial l}\left(\frac{1}{l}e^{\frac{\pi^2}{2\epsilon}|l|}\int d2\,{}_2\langle n(-l)|\,\pi_0^{(2)}\,|\Phi\rangle_2\right)\ . \end{aligned} \tag{A.6}$$

On the rightest hand side in the above equation, only the $\bar{\pi}_0\,|n(l')\rangle\,\vec{\chi}(l')$ component of $|\Phi\rangle$ provides a nonvanishing contribution because of the ghost zero-mode saturation, i.e.

$$\int d2\,{}_2\langle n(-l)|\,\pi_0^{(2)}\,|\Phi\rangle_2 = \int_{-\infty}^{\infty}dl'\int d2_2\langle n(-l)|\,\pi_0^{(2)}\bar{\pi}_0^{(2)}\,|n(l')\rangle_2\,\vec{\chi}(l')\ . \tag{A.7}$$

It is easy to show

$$\int d2\,{}_2\langle n(-l)|\,\pi_0^{(2)}\bar{\pi}_0^{(2)}\,|n(l')\rangle_2 = \frac{i|l|}{4\epsilon}\int d2\,{}_2\langle n(-l)|\,n(l')\rangle_2 = i\frac{\mathcal{N}^2}{4\epsilon}\,|l|\,l'\,\delta(l'-l)\ . \tag{A.8}$$

Plugging eqs. (A.8) and (A.7) into eq. (A.6), we obtain

$$\begin{aligned} &n(-l)\cdot Q_{\rm B}\Phi \\ &= e^{-\frac{\pi^2}{2\epsilon}|l|}l\frac{\partial}{\partial l}\left(e^{\frac{\pi^2}{2\epsilon}|l|}|l|\frac{\mathcal{N}^2}{4\epsilon}\vec{\chi}(l)\right) = \begin{cases} \dfrac{\mathcal{N}^2}{4\epsilon}l\left(\dfrac{\partial}{\partial l}+\dfrac{\pi^2}{2\epsilon}\right)(l\chi(l)) & \text{for } l>0 \\ -\dfrac{\mathcal{N}^2}{4\epsilon}l\left(\dfrac{\partial}{\partial l}-\dfrac{\pi^2}{2\epsilon}\right)(l\bar{\chi}(-l)) & \text{for } l<0 \end{cases}\ . \end{aligned} \tag{A.9}$$

Second, we consider the non-linear term $g\,|\Phi*\Phi\rangle$. Keeping the most dominant terms in the limit $\epsilon\to 0$, we find that

$$n(-l)\cdot(\Phi*\Phi) = \int d1d2d3d4\ \langle V_3(1,2,3)\,|\,\Phi\rangle_1\,|\Phi\rangle_2\,{}_4\langle n(-l)\,|R(3,4)\rangle$$

$$
\begin{aligned}
&= - \int d1d2d3 \, \langle V_3(1,2,3)| \, \Phi\rangle_1 \, |\Phi\rangle_2 \, |n(-l)\rangle_3 \\
&= - \int d1d2d3 \, \langle V_3^0(1,2,3)| \, C^{(r)}(\rho_I) \, |\Phi\rangle_1 \, |\Phi\rangle_2 \, |n(-l)\rangle_3 \\
&= - \int_{-\infty}^{\infty} dl_1 dl_2 \int d1d2d3 \, \langle V_3^0(1,2,3)| \Big[C^{(r)}(\rho_I) \bar{\pi}_0^{(1)} \, |n(l_1)\rangle_1 \, |n(l_2)\rangle_2 \, |n(-l)\rangle_3 \, \vec{\chi}(l_1) \vec{\phi}(l_2) \\
&\qquad\qquad + C^{(r)}(\rho_I) \bar{\pi}_0^{(2)} \, |n(l_1)\rangle_1 \, |n(l_2)\rangle_2 \, |n(-l)\rangle_3 \, \vec{\phi}(l_1) \vec{\chi}(l_2) \Big] \\
&\qquad + \cdots \, , \qquad\qquad \text{(A.10)}
\end{aligned}
$$

where r of $C^{(r)}$ can be any of $1,2,3$. In going from the third line to the fourth line in the above equation, we expand $|\Phi\rangle_{1,2}$ in terms of the complete basis defined in subsection 4.3. In doing so, we have used the following idempotency equations

$$
\int d3 \langle V_3^0(1,2,3)|n(-l)\rangle_3 \propto \int dl_1 \int dl_2 \delta(l_1 + l_2 - l) \, {}_1\langle n(l_1)| \, {}_2\langle n(l_2)| \frac{1}{\alpha_1 \alpha_2} \, , \qquad \text{(A.11)}
$$

for $|\alpha_1|, |\alpha_2| < |l|$, and

$$
\int drds \langle V_3^0(1,2,3)|n(l_r)\rangle_r |n(l_s)\rangle_s \propto {}_t\langle n(-l_r - l_s)| \frac{1}{\alpha_t} \, , \qquad \text{(A.12)}
$$

where $r,s,t \in \{1,2,3\}$ and $r > s, \ r \neq t, \ s \neq t$. These equations hold in the limit $\epsilon \to 0$ and can be proved by using the connection conditions satisfied by $\langle V_3^0|$. '$\cdots$' on the rightest hand side stands for the contributions from the component fields other than $\vec{\phi}(l)$ and $\vec{\chi}(l)$. One can easily see that these terms include one annihilation operator other than ϕ, χ and one creation operator other than $\bar{\phi}, \bar{\chi}$. We will ignore these contributions in the rest of this appendix. Combining the ghost number conservation with the above arguments, we obtain the last equality in eq. (A.10). Since we may choose an arbitrary r for $C^{(r)}$ on the three string vertex $\langle V_3^0|$ as mentioned above, we set $r=1$ in the first term and set $r=2$ in the second term on the rightest hand side in eq. (A.10). From the definition (4.19) of the state$|n(l)\rangle$ and the fact that the field $C(\rho_I)$ satisfy the Dirichlet boundary condition on the state $|B_0\rangle$, one can find that

$$
C(\rho_I) \, |n(l)\rangle = \mathcal{O}(\epsilon) \, , \qquad \text{(A.13)}
$$

and thus

$$
C(\rho_I) \, \bar{\pi}_0 \, |n(l)\rangle = |n(l)\rangle + \mathcal{O}(\epsilon) \, . \qquad \text{(A.14)}
$$

This implies that in the leading order of ϵ, eq. (A.10) becomes

$$
\begin{aligned}
&n(-l) \cdot (\Phi * \Phi) \qquad\qquad \text{(A.15)} \\
&= - \int_{-\infty}^{\infty} dl_1 dl_2 \int d1d2d3 \, \langle V_3^0(1,2,3)| \, n(l_1)\rangle_1 \, |n(l_2)\rangle_2 \, |n(-l)\rangle_3 \left(\vec{\chi}(l_1)\vec{\phi}(l_2) + \vec{\phi}(l_1)\vec{\chi}(l_2) \right) \\
&= -\pi^3 \frac{\left(4\pi^2\alpha'\right)^{\frac{p+1}{2}}}{\left(2\pi^2\alpha'\right)^{\frac{13}{2}}} \sqrt{\frac{V_{\mathrm{D}}}{V_{\mathrm{N}}}} \frac{\mathcal{N}^3}{\epsilon^3} \int_{-\infty}^{\infty} dl_1 dl_2 \, |l_1 l_2 l| \, \delta\left(l_1 + l_2 - l\right) \frac{1}{2} \left(\vec{\chi}(l_1)\vec{\phi}(l_2) + \vec{\phi}(l_1)\vec{\chi}(l_2) \right) \, .
\end{aligned}
$$

In the second equality in this equation, we have used the result obtained in Appendix B. We can further recast eq. (A.15) into

$$
\begin{aligned}
&n(-l)\cdot(\Phi*\Phi)\\
&=-\pi^3\frac{\left(4\pi^2\alpha'\right)^{\frac{p+1}{2}}}{\left(2\pi^2\alpha'\right)^{\frac{13}{2}}}\sqrt{\frac{V_{\rm D}}{V_{\rm N}}}\frac{\mathcal{N}^3}{\epsilon^3}\,|l|\int_{-\infty}^{\infty}dl_1\,\left|l_1(l-l_1)\right|\vec{\chi}(l_1)\vec{\phi}(l-l_1)\\
&=-\pi^3\frac{\left(4\pi^2\alpha'\right)^{\frac{p+1}{2}}}{\left(2\pi^2\alpha'\right)^{\frac{13}{2}}}\sqrt{\frac{V_{\rm D}}{V_{\rm N}}}\frac{\mathcal{N}^3}{\epsilon^3}\,|l|\times\\
&\quad\times\Bigg[\Theta(l)\left\{\int_0^l dl_1\,l_1(l-l_1)\chi(l_1)\phi(l-l_1)\right.\\
&\qquad\left.+\int_0^\infty dl_1\,l_1(l_1+l)\Big(\chi(l+l_1)\bar{\phi}(l_1)+\bar{\chi}(l_1)\phi(l+l_1)\Big)\right\}\\
&\qquad+\Theta(-l)\left\{\int_0^{-l} dl_1\,l_1(-l-l_1)\bar{\chi}(l_1)\bar{\phi}(-l-l_1)\right.\\
&\qquad\left.+\int_0^\infty dl_1\,l_1(l_1-l)\Big(\bar{\chi}(-l+l_1)\phi(l_1)+\chi(l_1)\bar{\phi}(l_1-l)\Big)\right\}\Bigg]\ ,
\end{aligned}
\tag{A.16}
$$

where Θ is the step function defined as $\Theta(x)=1$ for $x>0$ and $=0$ for $x<0$.

Combining eqs. (A.2), (A.9) and (A.16), we obtain eq. (4.28).

B. $\int d1d2d3\,\langle V_3^0(1,2,3)|\,n(l_1)\rangle_1\,|n(l_2)\rangle_2\,|n(-l)\rangle_3$

In this appendix, we will prove that the following relation holds in the leading order of ϵ:

$$
\begin{aligned}
&\int d1d2d3\,\left\langle V_3^0(1,2,3)\right|\,n(l_1)\rangle_1\,|n(l_2)\rangle_2\,|n(-l)\rangle_3\\
&\quad\simeq\frac{1}{2}\left(\pi^3\frac{\left(4\pi^2\alpha'\right)^{\frac{p+1}{2}}}{\left(2\pi^2\alpha'\right)^{\frac{13}{2}}}\sqrt{\frac{V_D}{V_N}}\right)\frac{\mathcal{N}^3}{\epsilon^3}|l_1l_2l|\delta(l_1+l_2-l)\ .
\end{aligned}
\tag{B.1}
$$

This equation was used in eq. (A.15) to derive eq. (4.28). Therefore, by proving eq. (B.1), we can complete the derivation of eq. (4.28) presented in the last appendix. From the definition (4.19), we find that we can prove the above equation by evaluating

$$
\int d'1d'2d'3\,\left\langle V_3^0(1,2,3)\right|\,B_0\rangle_1^\epsilon\,|B_0\rangle_2^\epsilon\,|B_0\rangle_3^\epsilon\ . \tag{B.2}
$$

Here we have introduced the integration measure $d'r$ for zero modes defined by removing the α dependence from dr given in eq. (3.7):

$$
d'r=(2\pi)^{-27}d^{26}p_r id\bar{\pi}_0^{(r)}d\pi_0^{(r)}\ . \tag{B.3}
$$

The amplitude (B.2) corresponds to the string diagram described in figure 2(a).

As was performed in ref. [24], we use conformal field theory (CFT) technique to calculate eq. (B.2). In the OSp invariant string theory, the ghost sector $(C,\bar{C})$ is described by the CFT with central charge -2. The full theory, which consists of the matter sector

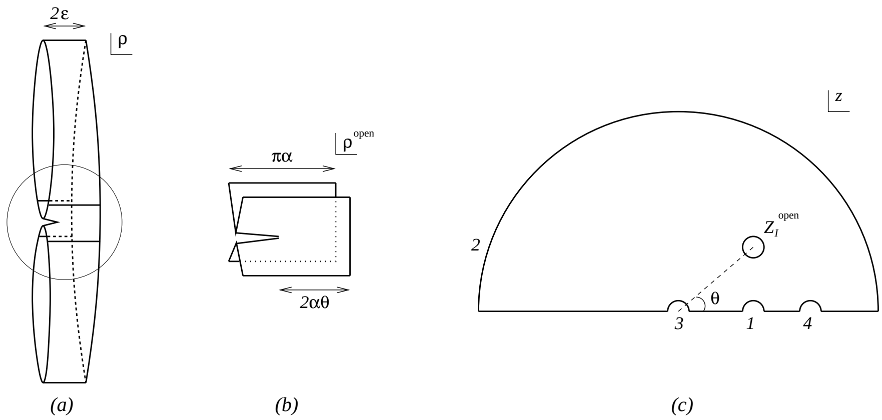

Figure 2: (a) The closed string 3-point diagram in the limit $\epsilon \to 0$. (b) The open string 4-point diagram. (c) The upper half z-plane after cutting out the circle around the interaction point Z_I and semicircles around the points Z_r.

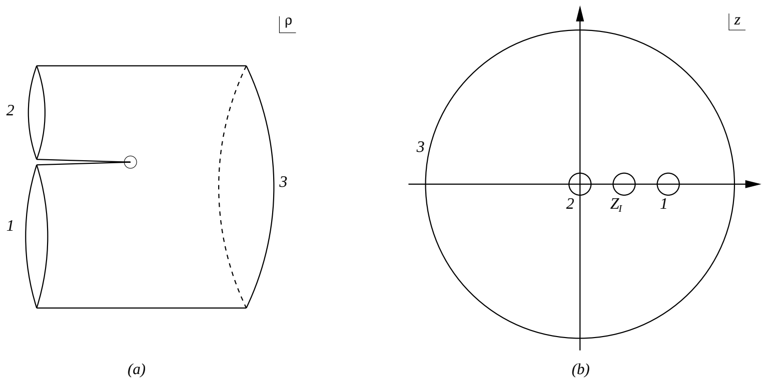

Figure 3: (a) The closed string 3-point diagram. (b) The complex z-plane after cutting out circles around the interaction point Z_I and the points Z_r.

X^μ in addition to the ghost sector, is therefore the CFT with total central charge $c = 24$. Since $c \neq 0$, the amplitudes of this system depend on the metric on the worldsheet. The CFT we are dealing with consists of free bosons and fermions. Therefore the metric dependence stems from the determinant of the Laplacian on the worldsheet. It can be given by evaluating the Liouville action on the worldsheet [28, 29].

The oscillator independent part of the three string vertex $\langle V_3^0|$ can be considered to be due to the contributions from the Liouville action. $\langle V_3^0|$ corresponds to the diagram depicted in figure 3(a). It is useful to pull this diagram (ρ-plane) back to the complex z-plane (figure 3(b)) through the Mandelstam mapping [15],

$$\rho(z) = \alpha_1 \ln(z - Z_1) + \alpha_2 \ln(z - Z_2) + \alpha_3 \ln(z - Z_3) \ . \tag{B.4}$$

We fix the $SL(2, \mathbb{C})$ gauge symmetry on the worldsheet by setting $Z_1 = 1$, $Z_2 = 0$ and $Z_3 = \infty$. The interaction point is at $Z_I = -\frac{\alpha_2}{\alpha_3}$, where we have $\frac{\partial \rho}{\partial z} = 0$. In order to avoid the singularity at the interaction point, we cut out a small circle of radius r_I around the interaction point in the ρ-plane. We also cut the points corresponding to incoming and outgoing strings at $\tau = \pm\infty$ by terminating each string at $\tau = \tau_r$ ($r = 1, 2, 3$). These correspond to cutting circles out of the z-plane centered on $z = Z_p$ with small radii ε_p ($p = 1, 2, 3, I$), as represented in figure 3(b).

Let us suppose that the ρ-plane is equipped with the flat metric:

$$ds^2 = d\rho d\bar{\rho} = e^{\phi} dz d\bar{z} \ , \quad \phi = \ln \left| \frac{\partial \rho}{\partial z} \right|^2 \ . \tag{B.5}$$

We will see that $\langle V_3^0|$ is constructed to reproduce the CFT amplitudes on the ρ-plane with this metric. Since the calculations are done on the z-plane, we should take care of the Liouville action for the Liouville field ϕ in eq. (B.5), because the determinant of the Laplacian is expressed as

$$\ln \det{}'\Delta\big|_{\phi} - \ln \det{}'\Delta\big|_{\phi=0} \sim -\frac{1}{48\pi} \left[\int d^2\sigma \partial_a \phi \partial^a \phi + 4 \int_{\partial \mathcal{M}} ds\, \hat{k} \phi \right] \ , \tag{B.6}$$

where s denotes the variable parametrizing the boundary of the worldsheet $\mathcal{M}$; $\hat{k}$ denotes the geodesic curvature of the boundary defined as

$$\hat{k} = n_b t^a \hat{\nabla}_a t^b \ , \tag{B.7}$$

where t^a is the unit vector tangential to the boundary while $n^a = -\frac{\epsilon^{ab}}{\sqrt{\hat{h}}} t_b$ is normal, and $\hat{\nabla}_a$ denotes the covariant derivative associated with the metric $ds^2 = dz d\bar{z}$.

The dependence of $\ln \det{}'\Delta\big|_{\phi=0}$ on ε_p was calculated in Appendix 11.A of ref. [29] [8]:

$$\ln \det{}'\Delta\big|_{\phi=0} \sim -\frac{1}{3} \sum_p \ln \varepsilon_p \ , \tag{B.8}$$

where $\sum_p$ denotes the sum over all the values of I and r. By exploring the Mandelstam mapping (B.4) near the cuts, we find that ε_p depend on α_r as follows:

$$\ln \varepsilon_r \sim -\tau_r + \frac{\hat{\tau}_0}{\alpha_r} \quad (r = 1, 2, 3) \ , \qquad \ln \varepsilon_I \sim \frac{1}{2} (\ln 2 r_I - \ln |c_I|) \ , \tag{B.9}$$

where

$$c_I = \frac{\partial^2 \rho}{\partial z^2}\bigg|_{z = Z_I} = \frac{{\alpha_3}^3}{\alpha_1 \alpha_2} \ . \tag{B.10}$$

Using these results, we obtain

$$\ln \det{}'\Delta \sim \frac{1}{6} \sum_{r=1}^{3} \frac{\hat{\tau}^0}{\alpha_r} + \frac{1}{12} \sum_{r=1}^{3} \ln |\alpha_r| \ . \tag{B.11}$$

Thus we find that the determinant factor depends on α_r in the following way,

$$\left(\det{}'\Delta\right)^{-\frac{c}{2}} = \left(\det{}'\Delta\right)^{-12} \propto |\mu(1, 2, 3)|^2 \frac{1}{|\alpha_1 \alpha_2 \alpha_3|} \ . \tag{B.12}$$

[8] Eq. (B.8) is twice eq.(11.A.26) in ref. [29] because we are dealing with the closed string case.

This reproduces the factor appearing in the three string vertex $\langle V_3^0|$ given in eq. (3.25), except for the fact that we should take the absolute value of the factor $\alpha_1\alpha_2\alpha_3$. Thus we see that roughly speaking, $\langle V_3^0|$ is constructed to reproduce the CFT amplitudes on the ρ-plane with the metric (B.5). Precisely speaking, $\mathrm{sgn}\,(\alpha_1\alpha_2\alpha_3)\,\langle V_3^0|$ corresponds to figure 3(a).

Now we would like to calculate eq. (B.2) in the limit $\epsilon \to 0$. For this purpose, it is convenient to see the string diagram figure 2(a) from the point of view of the dual open string channel. In this channel, one can regard the worldsheet as being swept by four open strings interacting via mid-point type interaction figure 2(b). In the limit $\epsilon \to 0$, such open strings propagate through very long proper time and thus the most dominant contribution comes from the propagations of the open string tachyons. The propagator contributes the factor

$$e^{\frac{\pi^2}{\epsilon}\max(|\alpha_1|,|\alpha_2|,|\alpha_3|)} \,. \tag{B.13}$$

Let us evaluate the determinant of the Laplacian on the worldsheet depicted by figure 2(b). The Mandelstam mapping from the upper half z-plane into the open string 4-point ρ^{open}-plane is

$$\begin{aligned}\rho^{\mathrm{open}}(z) = \alpha_1^{\mathrm{open}}\ln(z-Z_1^{\mathrm{open}}) + \alpha_2^{\mathrm{open}}\ln(z-Z_2^{\mathrm{open}})\\ +\alpha_3^{\mathrm{open}}\ln(z-Z_3^{\mathrm{open}}) + \alpha_4^{\mathrm{open}}\ln(z-Z_4^{\mathrm{open}}) \,.\end{aligned} \tag{B.14}$$

For the worldsheet described in figure 2(b), we should choose

$$\alpha_1^{\mathrm{open}} = \alpha_2^{\mathrm{open}} = -\alpha_3^{\mathrm{open}} = -\alpha_4^{\mathrm{open}} = \frac{2\epsilon}{\pi} \equiv \alpha \,. \tag{B.15}$$

Let us choose $Z_r^{\mathrm{open}} = (1,\infty,0,x)$ $(r=1,2,3,4)$. Here we should take $x>1$ to treat the worldsheet that we are considering. The interaction point is

$$Z_I^{\mathrm{open}} = 1 + i\sqrt{x-1} \,, \tag{B.16}$$

which is a solution of $\left.\frac{d\rho^{\mathrm{open}}}{dz}\right|_{z=Z_I^{\mathrm{open}}} = 0$. We introduce the parameter θ defined by

$$\cos\theta = \frac{1}{\sqrt{x}} \,, \quad \sin\theta = \frac{\sqrt{x-1}}{\sqrt{x}} \,. \tag{B.17}$$

In terms of θ, the interaction point is described by

$$Z_I^{\mathrm{open}} = \frac{1}{\cos\theta}e^{i\theta} \,, \quad \rho_I^{\mathrm{open}} \equiv \rho^{\mathrm{open}}(Z_I^{\mathrm{open}}) = 2\alpha(\ln\cos\theta - i\theta) \,. \tag{B.18}$$

$\theta = \frac{\pi}{4}$ in our case, but let us treat θ as a free parameter in order to compare with the results of [12]. In order to avoid the singularities, we excise small circles around the interaction point and external strings, as shown in figure 2(c). We define τ_r^{open} accordingly. By using the metric (B.5), we find that the moduli dependence of the determinant of the Laplacian becomes

$$\begin{aligned}\ln\det{}'\Delta &\sim -\frac{1}{48\pi}\left[\int d^2\sigma\partial_a\phi\partial^a\phi + 4\int_{\partial\mathcal{M}} ds\, k\phi\right] + \frac{1}{6}\sum_{r=1}^{4}\ln\varepsilon_r^{\mathrm{open}} + \frac{1}{3}\ln\varepsilon_I^{\mathrm{open}}\\ &\sim \frac{1}{4}\ln(\alpha\cos\theta\sin\theta) \,.\end{aligned} \tag{B.19}$$

Here we have used

$$\begin{aligned}
\ln \varepsilon_1^{\text{open}} &\sim -\tau_1^{\text{open}} + \frac{\hat{\tau}_0^{\text{open}}}{\alpha} + \ln(x-1) \ , \\
\ln \varepsilon_2^{\text{open}} &\sim -\tau_2^{\text{open}} + \frac{\hat{\tau}_0^{\text{open}}}{\alpha} - \ln x \ , \\
\ln \varepsilon_3^{\text{open}} &\sim -\tau_3^{\text{open}} - \frac{\hat{\tau}_0^{\text{open}}}{\alpha} \ , \\
\ln \varepsilon_4^{\text{open}} &\sim -\tau_4^{\text{open}} - \frac{\hat{\tau}_0^{\text{open}}}{\alpha} - \ln x + \ln(x-1) \ , \\
\ln \varepsilon_I^{\text{open}} &\sim \frac{1}{2}(\ln 2r_I^{\text{open}} - \ln |c_I^{\text{open}}|) \ ,
\end{aligned} \tag{B.20}$$

with

$$\begin{aligned}
c_I^{\text{open}} &= \frac{2i\alpha}{\sqrt{x-1}(1+i\sqrt{x-1})^2} = \frac{2i\alpha\cos^3\theta}{e^{2i\theta}\sin\theta} \ , \\
\hat{\tau}_0^{\text{open}} &= 2\alpha \ln\cos\theta \ .
\end{aligned} \tag{B.21}$$

Combining eqs. (B.13) and (B.19), we find that the amplitude corresponding to the pants diagram in the limit $\epsilon \to 0$ depends on α_r, α and θ as

$$\int d'1d'2d'3 \left\langle V_3^0(1,2,3)\right| B_0\rangle_1^{(\pi-2\theta)\alpha} |B_0\rangle_2^{(\pi-2\theta)\alpha} |B_0\rangle_3^{2\theta\alpha} \propto \frac{e^{\frac{\pi^2}{\epsilon}\max(|\alpha_1|,|\alpha_2|,|\alpha_3|)}}{\alpha^3\sin^3\theta\cos^3\theta} . \tag{B.22}$$

In ref. [12], the authors computed this quantity in the limit $\theta \to 0$, by using the Cremmer-Gervais identity [16]. By comparing our result with theirs, it is straightforward to determine the overall constant, and we obtain

$$\begin{aligned}
&\int d'1d'2d'3 \left\langle V_3^0(1,2,3)\right| B_0\rangle_1^{\epsilon} |B_0\rangle_2^{\epsilon} |B_0\rangle_3^{\epsilon} \\
&\simeq \frac{1}{2}\left(\pi^3 \frac{(4\pi^2\alpha')^{\frac{p+1}{2}}}{(2\pi^2\alpha')^{\frac{13}{2}}}\sqrt{\frac{V_D}{V_N}}\right)\frac{\mathcal{N}^3}{\epsilon^3}\frac{|\alpha_1\alpha_2\alpha_3|}{\alpha_1\alpha_2\alpha_3} e^{\frac{\pi^2}{\epsilon}\max(|\alpha_1|,|\alpha_2|,|\alpha_3|)} \ .
\end{aligned} \tag{B.23}$$

When we substitute eq. (B.23) into eq. (B.1), the exponential factor on the right hand side in this equation is canceled by the regularization factors $e^{-\frac{\pi^2}{2\epsilon}|l_r|}$ $(r=1,2,3)$ in $|n(l_r)\rangle_r$. It is because $\max(|\alpha_1|,|\alpha_2|,|\alpha_3|) = \frac{1}{2}(|\alpha_1|+|\alpha_2|+|\alpha_3|)$. Finally, carrying out the α_r integration, we obtain eq. (B.1).

A comment is in order. In eq. (B.1), there is a factor $|l_1 l_2 l|$. The absolute value originates from the one in eq. (B.12). Taking the absolute value is necessary to be consistent with

$$\begin{aligned}
&\left(\int d1d2d3\langle V_3^0(1,2,3)|n(l_1)\rangle_1|n(l_2)\rangle_2|n(l_3)\rangle_3\right)^\dagger \\
&= \int d1d2d3\langle V_3^0(1,2,3)|n(-l_1)\rangle_1|n(-l_2)\rangle_2|n(-l_3)\rangle_3 \ ,
\end{aligned} \tag{B.24}$$

which can be easily proved.

References

[1] L. Rastelli, A. Sen and B. Zwiebach, *String field theory around the tachyon vacuum*, *Adv. Theor. Math. Phys.* **5** (2002) 353 [hep-th/0012251];
for reviews and references, see W. Taylor and B. Zwiebach, *D-branes, tachyons and string field theory*, hep-th/0311017;
A. Sen, *Tachyon dynamics in open string theory*, *Int. J. Mod. Phys.* **A 20** (2005) 5513 [hep-th/0410103].

[2] K. Hashimoto and H. Hata, *D-brane and gauge invariance in closed string field theory*, *Phys. Rev.* **D 56** (1997) 5179 [hep-th/9704125].

[3] T. Asakawa, S. Kobayashi and S. Matsuura, *Closed string field theory with dynamical D-brane*, *JHEP* **10** (2003) 023 [hep-th/0309074].

[4] V. Fateev, A.B. Zamolodchikov and A.B. Zamolodchikov, *Boundary Liouville field theory, I. Boundary state and boundary two-point function*, hep-th/0001012;
A.B. Zamolodchikov and A.B. Zamolodchikov, *Liouville field theory on a pseudosphere*, hep-th/0101152;
J. Teschner, *Remarks on Liouville theory with boundary*, hep-th/0009138;
B. Ponsot and J. Teschner, *Boundary Liouville field theory: boundary three point function*, *Nucl. Phys.* **B 622** (2002) 309 [hep-th/0110244].

[5] M. Fukuma and S. Yahikozawa, *Nonperturbative effects in noncritical strings with soliton backgrounds*, *Phys. Lett.* **B 396** (1997) 97 [hep-th/9609210]; *Combinatorics of solitons in noncritical string theory*, *Phys. Lett.* **B 393** (1997) 316 [hep-th/9610199].

[6] M. Fukuma, H. Kawai and R. Nakayama, *Continuum Schwinger-Dyson equations and universal structures in two-dimensional quantum gravity*, *Int. J. Mod. Phys.* **A 6** (1991) 1385;
R. Dijkgraaf, H.L. Verlinde and E.P. Verlinde, *Loop equations and Virasoro constraints in nonperturbative 2D quantum gravity*, *Nucl. Phys.* **B 348** (1991) 435.

[7] M. Hanada et al., *Loops versus matrices: the nonperturbative aspects of noncritical string*, *Prog. Theor. Phys.* **112** (2004) 131 [hep-th/0405076].

[8] N. Ishibashi and H. Kawai, *String field theory of noncritical strings*, *Phys. Lett.* **B 314** (1993) 190 [hep-th/9307045];
A. Jevicki and J.P. Rodrigues, *Loop space hamiltonians and field theory of noncritical strings*, *Nucl. Phys.* **B 421** (1994) 278 [hep-th/9312118].

[9] W. Siegel, *Covariantly second quantized string*, *Phys. Lett.* **B 142** (1984) 276.

[10] M. Kaku and K. Kikkawa, *The field theory of relativistic strings, pt. 1. Trees*, *Phys. Rev.* **D 10** (1974) 1110; *The field theory of relativistic strings. pt. 2. Loops and pomerons*, *Phys. Rev.* **D 10** (1974) 1823.

[11] T. Kugo, *Covariantized light cone string field theory*, in *Quantum Mechanics of Fundamental Systems 2*, C. Teitelboim and J. Zanelli eds., Plenum Publishing Corporation, 1989, chap. 11.

[12] I. Kishimoto, Y. Matsuo and E. Watanabe, *Boundary states as exact solutions of (vacuum) closed string field theory*, *Phys. Rev.* **D 68** (2003) 126006 [hep-th/0306189]; *A universal nonlinear relation among boundary states in closed string field theory*, *Prog. Theor. Phys.* **111** (2004) 433 [hep-th/0312122].

[13] T. Yoneya, *Extended fermion representation of multi-charge 1/2-BPS operators in AdS/CFT: towards field theory of D-branes*, *JHEP* **12** (2005) 028 [hep-th/0510114].

JHEP05(2006)029

[14] T. Okuda and T. Takayanagi, *Ghost D-branes*, *JHEP* **03** (2006) 062 [hep-th/0601024].

[15] S. Mandelstam, *Interacting string picture of dual resonance models*, *Nucl. Phys.* **B 64** (1973) 205.

[16] E. Cremmer and J.L. Gervais, *Infinite component field theory of interacting relativistic strings and dual theory*, *Nucl. Phys.* **B 90** (1975) 410.

[17] W. Siegel and B. Zwiebach, *Gauge string fields from the light cone*, *Nucl. Phys.* **B 282** (1987) 125; *Closed string fields via IOSP(1,1/2)*, *Phys. Lett.* **B 184** (1987) 325; *OSP(1,1/2) string field theory with the world sheet metric*, *Nucl. Phys.* **B 288** (1987) 332;
L. Baulieu, W. Siegel and B. Zwiebach, *New coordinates for string fields*, *Nucl. Phys.* **B 287** (1987) 93.

[18] A.K.H. Bengtsson and N. Linden, *Interacting covariant open bosonic strings from the light cone $j(i-)$*, *Phys. Lett.* **B 187** (1987) 289.

[19] A. Neveu and P.C. West, *String lengths in covariant string field theory and OSP(26,2/2)*, *Nucl. Phys.* **B 293** (1987) 266.

[20] S. Uehara, *On the covariantized light cone string field theory*, *Phys. Lett.* **B 190** (1987) 76; *On the 'covariantized light cone' string field theory, 2*, *Phys. Lett.* **B 196** (1987) 47.

[21] T. Kawano, *Dilaton condensation in the covariantized light cone closed string field theory*, *Prog. Theor. Phys.* **88** (1992) 1181.

[22] H. Hata, K. Itoh, T. Kugo, H. Kunitomo and K. Ogawa, *Gauge string field theory for torus compactified closed string*, *Prog. Theor. Phys.* **77** (1987) 443.

[23] M. Maeno and H. Takano, *Derivation of the cocycle factor of vertex in closed bosonic string field theory on torus*, *Prog. Theor. Phys.* **82** (1989) 829.

[24] I. Kishimoto and Y. Matsuo, *Cardy states as idempotents of fusion ring in string field theory*, *Phys. Lett.* **B 590** (2004) 303 [hep-th/0402107]; *Cardy states, factorization and idempotency in closed string field theory*, *Nucl. Phys.* **B 707** (2005) 3 [hep-th/0409069].

[25] K. Murakami and T. Nakatsu, *Open Wilson lines as states of closed string*, *Prog. Theor. Phys.* **110** (2003) 285 [hep-th/0211232].

[26] H. Hata, K. Itoh, T. Kugo, H. Kunitomo and K. Ogawa, *Covariant string field theory, 2*, *Phys. Rev.* **D 35** (1987) 1318.

[27] W. Fischler and L. Susskind, *Dilaton tadpoles, string condensates and scale invariance*, *Phys. Lett.* **B 171** (1986) 383;
Dilaton tadpoles, string condensates and scale invariance, 2, *Phys. Lett.* **B 173** (1986) 262.

[28] O. Alvarez, *Theory of strings with boundaries: fluctuations, topology and quantum geometry*, *Nucl. Phys.* **B 216** (1983) 125.

[29] M.B. Green, J.H. Schwarz and E. Witten, *Superstring theory, vol. 2. Loop amplitudes, anomalies and phenomenology*, Cambridge University Press, 1987.